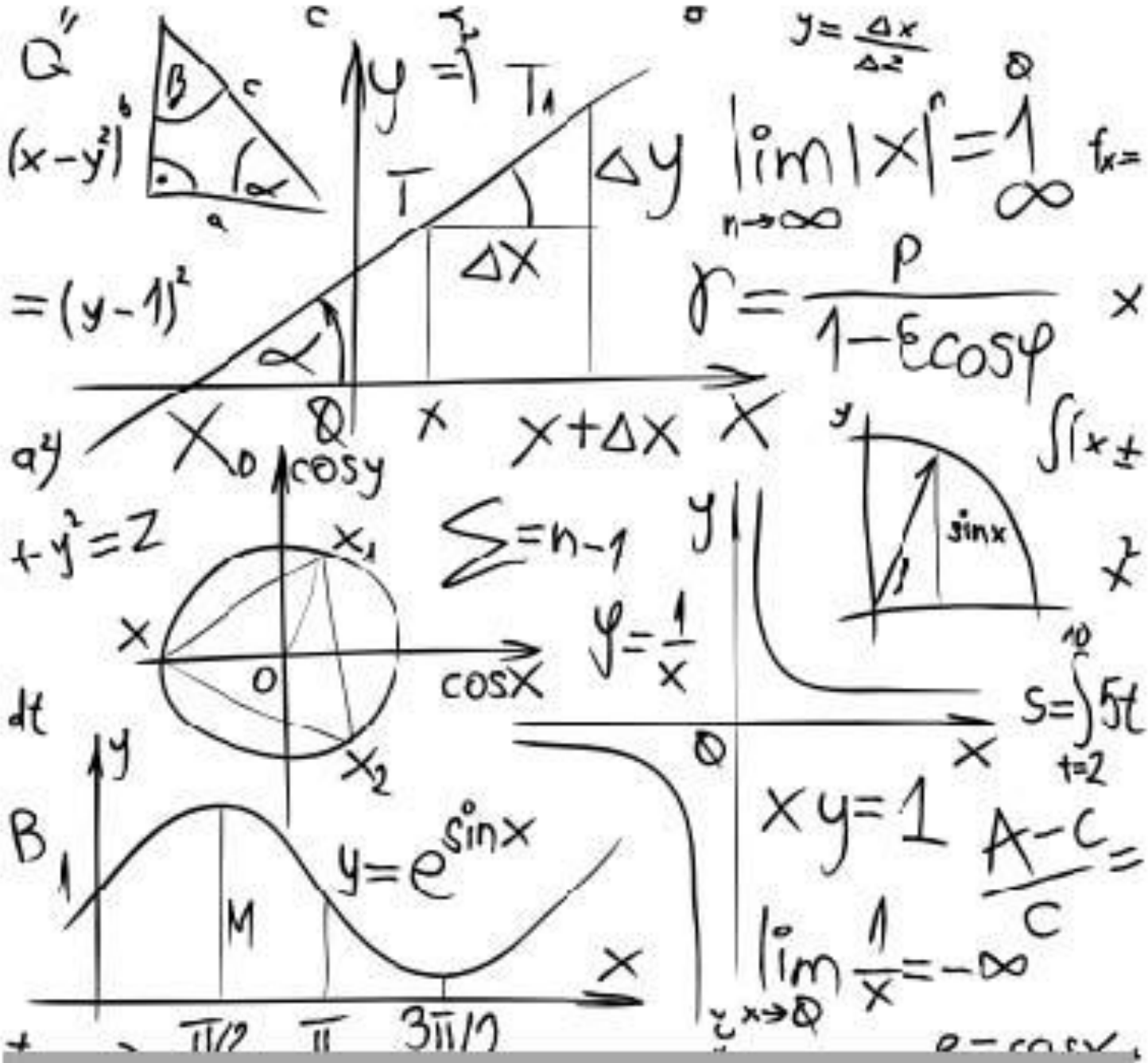

Ripley St Thomas

A-Level Maths

Bridging Provision

Introduction

In order to prepare for taking A-level maths next year, you need to ensure you are fluent in all aspect of GCSE, particularly algebra and those included in this booklet, and can approach problems with a confident, logical mind-set.

You are expected to use this booklet to fill gaps in your knowledge, build confidence and ensure you start A-level mathematics fully prepared. The knowledge and skills you find here, are essential for success in A-Level Mathematics.

The last 15 pages of this booklet are your first transition piece. This is **compulsory** and **must** be handed in during your first A-level mathematics lesson.

Success in A-Level Mathematics relies on an excellent attitude to learning and commitment to your studies. If you are struggling with any of the content in this booklet, you must use the video links included to brush up on these key skills. The videos stated are accessible on www.hegartymaths.com (if you have access to this through your current school) OR www.corbettmaths.com (for everyone).

Content

Looking for something extra? Try Mr Hegarty's live A-level prep lessons, also available on YouTube (accessible for all)!

Self-Assessment Grid

Complete the 'RAG' table and use this as a starting point to direct your independent study. You can then revisit this and update your self-assessment as you grow in confidence.

			
Quadratics			
Factorise a quadratic expression where a = 1			
Factorise a 'DOTS' type quadratic expression			
Factorise a quadratic expression where a ≠ 1			
Solve a quadratic equation by factorising			
Write a quadratic expression in completed square form			
Solve a quadratic equation by completing the square			
Solve a quadratic equation using the quadratic formula			
Quadratic Graphs			
Complete a table of values and plot a quadratic graph			
Sketch a quadratic graph given its equation by:			
• Finding the x-intercept(s) (if any)			
• Finding the y-intercept			
• Finding the coordinates of the vertex by writing in completed square form			
Write down the equation of a quadratic given the graph			
Linear Graphs			
Write the equation for/draw a straight line using the gradient and intercept			
Find the gradient and y-intercept of a line by rearranging its equation			
Find the equation of a line using gradient and coordinates			
Find the equation of a perpendicular line			
Surds and Indices			
Apply the Index Laws with numbers or algebra			
Work with negative and fractional indices			
Manipulate expressions containing surds (including simplifying surds, expanding brackets containing surds and rationalising a denominator)			
Simultaneous Equations			
Solve linear simultaneous equations by elimination			
Solve linear simultaneous equations by substitution			
Solve simultaneous equations where one is linear and one quadratic or a circle			
Solve simultaneous equations graphically			
Rearranging Equations			
Change the subject of a formula			
Inequalities			
Solve linear inequalities			
Solve quadratic inequalities			
Plot a region using inequalities on a graph			
Trigonometry			
Find sides and angles in triangles using SOHCAHTOA in 2d and 3d			
Find angles, sides and areas in triangles using the Sine, Cosine and Area rules			

Bridging Unit 1 – Quadratics

You need to be able to confidently:

- Factorise a quadratic expression where a = 1
- Factorise a 'DOTS' type quadratic expression
- Factorise a quadratic expression where a ≠ 1
- Solve a quadratic equation by factorising
- Write a quadratic expression in completed square form
- Solve a quadratic equation by completing the square

1. Factorise a quadratic expression where a = 1

hegartymaths

Watch clip number #223

Or Corbett Maths #118

Practice Exercise 1.1

Complete these questions on file paper.
Factorise these expressions:

1. (a) $x^2 + 7x + 12$ (b) $x^2 + 6x + 8$ (c) $x^2 + 5x + 6$ (d) $x^2 + 8x + 7$
   (e) $x^2 + 4x + 4$ (f) $x^2 + 8x + 15$ (g) $x^2 + 6x + 9$ (h) $x^2 + 11x + 28$

2. (a) $x^2 + x - 12$ (b) $x^2 + 5x - 6$ (c) $x^2 + 3x - 10$ (d) $x^2 + 3x - 4$
   (e) $x^2 + 2x - 48$ (f) $x^2 + 4x - 32$ (g) $x^2 + 2x - 35$ (h) $x^2 + 8x - 33$

3. (a) $x^2 - 3x - 10$ (b) $x^2 - x - 20$ (c) $x^2 - 6x - 27$ (d) $x^2 - 2x - 3$
   (e) $x^2 - x - 12$ (f) $x^2 - 4x - 12$ (g) $x^2 - 4x - 21$ (h) $x^2 - 6x - 55$

4. (a) $x^2 - 6x + 9$ (b) $x^2 - 9x + 20$ (c) $x^2 - 9x + 14$ (d) $x^2 - 13x + 22$

(e) $x^2 - 9x + 8$ (f) $x^2 - 12x + 32$ (g) $x^2 - 15x + 36$ (h) $x^2 - 14x + 48$

5. (a) $x^2 - 9x + 8$ (b) $x^2 + 24x + 23$ (c) $x^2 - 5x - 14$ (d) $x^2 - 7x + 12$

(e) $x^2 + 12x + 36$ (f) $x^2 - 2x - 63$ (g) $x^2 + 14x + 24$ (h) $x^2 + 17x + 60$

Mark your work.

2. Factorise a 'DOTS' quadratic expression

hegartymaths

Watch clip number #224

Or Corbett Maths #120

Practice Exercise 1.2
Complete these questions on file paper.
Factorise these expressions:

1. (a) $x^2 - 25$ (b) $y^2 - 49$ (c) $w^2 - 100$ (d) $x^2 - 4$

(e) $c^2 - 64$ (f) $x^2 - 1$ (g) $x^2 - 900$ (h) $y^2 - 9$

(i) $16 - x^2$ (j) $1 - y^2$ (k) $81 - x^2$ (l) $144 - h^2$

(m) $x^2 - y^2$ (n) $a^2 - c^2$ (o) $9x^2 - 25$ (p) $4y^2 - 1$

2. (a) $x^4 - 1$ (b) $y^4 - 16$ (c) $a^4 - 25$ (d) $x^4 - y^4$

(e) $h^2 - p^4$ (f) $16x^4 - 49$ (g) $y^6 - 36$ (h) $x^6 - 64$

(i) $81p^4 - x^6$ (j) $144x^8 - 1$

Mark your work.

3. Factorise a quadratic where a ≠ 1

Watch clip number #225

Or Corbett Maths #119

Practice Exercise 1.3

Complete these questions on file paper.

Factorise these expressions:

1. (a) $2x^2 + 7x + 5$ (b) $2x^2 + 11x + 15$ (c) $2x^2 + 9x + 10$

   (d) $3x^2 + 13x + 4$ (e) $3x^2 + 4x + 1$ (f) $3x^2 + 8x + 4$

2. (a) $3x^2 + x - 4$ (b) $7x^2 + 20x - 3$ (c) $2x^2 - 13x + 15$

   (d) $3x^2 - 17x + 10$ (e) $3x^2 - 16x - 12$ (f) $3x^2 - x - 4$

3. (a) $6x^2 + 13x + 6$ (b) $9x^2 + 9x + 2$ (c) $6x^2 + 13x + 2$

   (d) $8x^2 + 41x + 5$ (e) $9x^2 + 6x + 1$ (f) $8x^2 + 26x + 15$

4. (a) $9x^2 - 12x - 5$ (b) $4x^2 - 4x - 3$ (c) $4x^2 - 11x + 6$

   (d) $6x^2 - 7x + 2$ (e) $10x^2 - 91x + 9$ (f) $4x^2 + 25x - 56$

Mark your work.

4. Solve a quadratic equation by factorising

Watch clip number #230

Or Corbett Maths #266

Practice Exercise 1.4
Complete these questions on file paper.
Solve these equations by factorising:

1. (a) $x^2 + 6x + 8 = 0$ (b) $x^2 + 7x + 12 = 0$ (c) $y^2 + 7y + 10 = 0$
   (d) $y^2 + 3y - 4 = 0$ (e) $x^2 - 2x - 8 = 0$ (f) $m^2 - 7m + 12 = 0$
   (g) $y^2 - 10y + 25 = 0$ (h) $y^2 - 4y - 45 = 0$ (i) $x^2 - x - 56 = 0$

2. (a) $x^2 - 9 = 0$ (b) $y^2 - 100 = 0$ (c) $w^2 - 1 = 0$
   (d) $k^2 - 144 = 0$ (e) $x^2 - 64 = 0$ (f) $c^2 - 0.25 = 0$

3. (a) $x^2 + 2x = -1$ (b) $y^2 + 8y + 10 = 3$ (c) $x^2 = 7x - 12$
   (d) $y^2 + 6y + 15 = 3 - 7y$ (e) $x^2 - x - 8 = 2x + 2$ (f) $2x^2 - 14x + 49 = x^2$

Mark your work.

5. Write a quadratic in completed square form

Watch clip number #235-237

Or Corbett Maths #10

Practice Exercise 1.5
Complete these questions on file paper.
Write these expressions in completed square form:

1. (a) $x^2 + 8x + 1$ (b) $x^2 + 10x + 3$ (c) $x^2 + 2x - 1$
   (d) $x^2 - 6x - 10$ (e) $x^2 - 4x - 13$ (f) $x^2 - 12x + 3$
   (g) $x^2 + 14x + 3$ (h) $x^2 - 2x - 15$ (i) $x^2 + 4x - 11$
   (j) $x^2 + x - 8$ (k) $x^2 + 3x + 1$ (l) $x^2 - 7x - 2$

2. (a) $2x^2 + 8x + 2$ (b) $2x^2 + 12x - 3$ (c) $3x^2 - 12x + 2$

(d) $4x^2 + 12x - 5$ (e) $2x^2 - 3x - 5$ (f) $5x^2 - 20x + 30$

Mark your work.

6. Solve a quadratic equation by completing the square

Watch clip number #238-239

Or Corbett Maths #267a

Practice Exercise 1.6
Complete these questions on file paper.
Solve these equations by completing the square:

1. (a) $x^2 + 6x + 8 = 0$ (b) $x^2 + 10x + 24 = 0$ (c) $x^2 + 14x + 40 = 0$

(d) $x^2 - 4x - 45 = 0$ (e) $x^2 - 12x + 35 = 0$ (f) $x^2 - 2x - 3 = 0$

2. (a) $x^2 + 5x + 4 = 0$ (b) $x^2 - 3x - 18 = 0$ (c) $x^2 + x - 12 = 0$

(d) $x^2 - 7x + 12 = 0$ (e) $x^2 - 11x + 24 = 0$ (f) $x^2 - 7x - 30 = 0$

3. Write your answers in surd form where appropriate:

(a) $x^2 + 4x - 3 = 0$ (b) $x^2 + 6x - 10 = 0$ (c) $x^2 - 2x - 5 = 0$

(d) $x^2 - 10x + 1 = 0$ (e) $x^2 + 8x + 3 = 0$ (f) $x^2 - 8x - 22 = 0$

4. (a) $5x^2 + 30x - 10 = 0$ (b) $2x^2 + 7x + 3 = 0$ (c) $3x^2 + 12x - 2 = 0$

(d) $2x^2 - 3x - 7 = 0$ (e) $5x^2 + 2x - 8 = 0$ (f) $10x^2 - 2x - 1 = 0$

Mark your work.

7. Solve a quadratic equation using the quadratic formula

Watch clip number #241-242

Or Corbett Maths #267

Practice Exercise 1.7

Complete these questions on file paper.

Solve these equations using the quadratic formula:

1. (a) $x^2 + 5x + 1 = 0$ (b) $2x^2 + 7x + 2 = 0$ (c) $4x^2 + 8x + 3 = 0$

   (d) $x^2 + 2x - 4 = 0$ (e) $3x^2 + 4x - 5 = 0$ (f) $2x^2 + 5x - 10 = 0$

   (g) $x^2 - 4x + 2 = 0$ (h) $7x^2 - 6x + 1 = 0$ (i) $3x^2 - 10x + 4 = 0$

   (j) $x^2 - x - 11 = 0$ (k) $x^2 - 6x - 20 = 0$ (l) $2x^2 - x - 9 = 0$

2. (a) $x^2 + 7x = 20$ (b) $2x^2 = 9x + 40$ (c) $3x^2 = 10 - 2x$

   (d) $x^2 - 8 = x$ (e) $7x = 13 - x^2$ (f) $4x^2 - 9 = 2x^2 + 4x$

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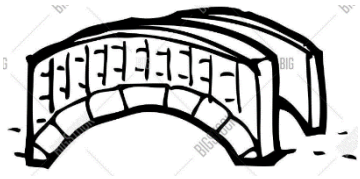


Bridging Unit 2 – Quadratic Graphs

You need to be able to confidently:

- Complete a table of values and plot a quadratic graph
- Sketch a quadratic graph given its equation by
  - Finding the x-intercept(s) (if any)
  - Finding the y-intercept
  - Finding the coordinates of the vertex by writing in completed square form
- Write down the equation of a quadratic given the graph

1. Complete a table of values and plot a quadratic graph

Watch clip number #251

Or Corbett Maths #264

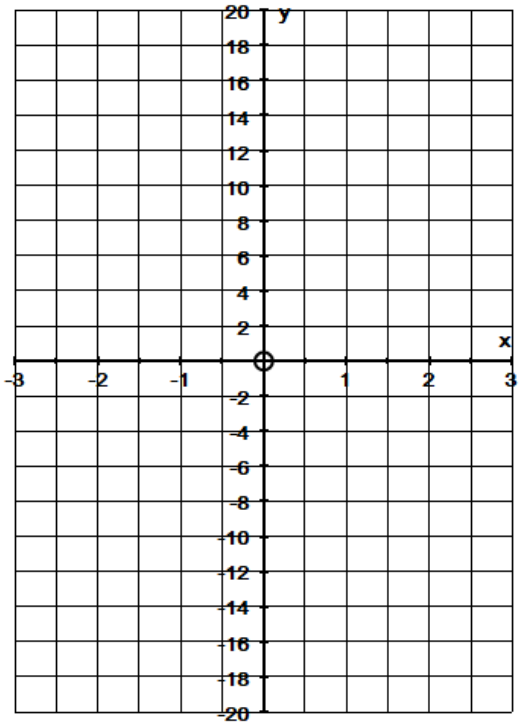
Practice Exercise 2.1

Print out the next page and answer the questions on it.

- If you use a calculator to work out the values, remember that when x is negative you must put brackets around it when you square it, i.e. $(-3)^2$.
- Make sure that you plot and draw the graphs in pencil.
- If your graph is not a smooth U or ∩ shape then at least one of your values must be incorrect or you have plotted something incorrectly, so look for your mistake and correct it.
- Each graph should be symmetrical, but not necessarily in the y-axis (and remember that you are only plotting a small section of it).

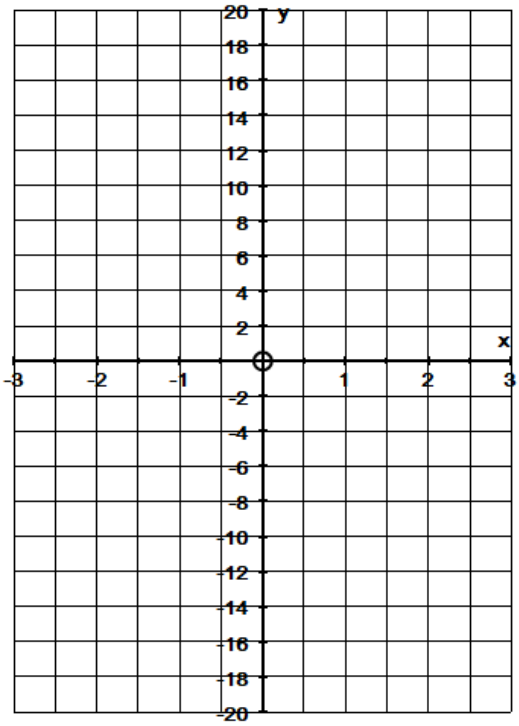
1. $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
y							



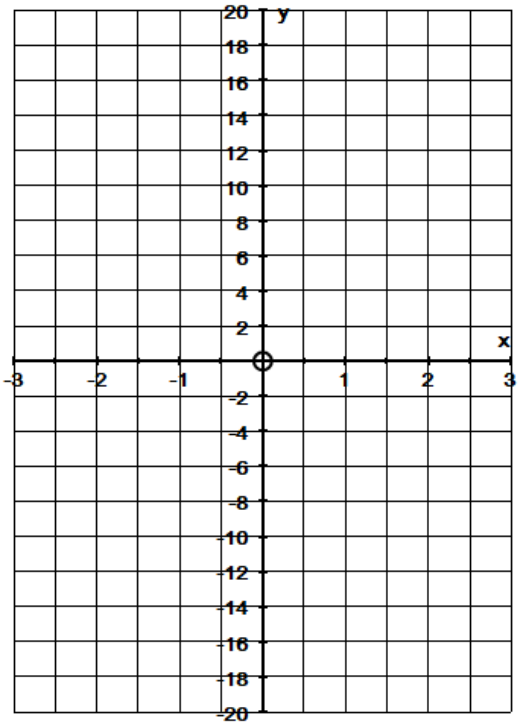
2. $y = x^2 + 2x$

x	-3	-2	-1	0	1	2	3
y							



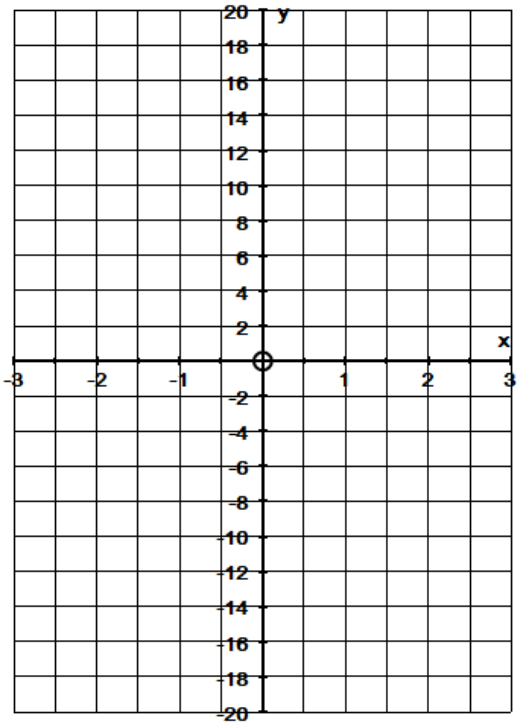
3. $y = 5x - x^2$

x	-3	-2	-1	0	1	2	3
y							



4. $y = x^2 + 4x + 7$

x	-3	-2	-1	0	1	2	3
y							



Mark your work.

2. Sketch a quadratic graph given its equation

This previous section was about plotting an accurate quadratic curve on numbered coordinate axes. What we are talking about here is *sketching the correct shape* of a quadratic graph, without scales on the axes but *labelling the significant points* with their coordinates (the intercepts and turning point).

Watch clip number #257

Or Corbett Maths #265

Practice Exercise 2.2

Complete these questions on **squared** paper.
Draw your graphs in pencil, and axes with a ruler.

For each question show that you have:

a. Decided whether the graph is U-shaped (if the coefficient of x^2 is positive) or ∩–shaped (if the coefficient of x^2 is negative).
b. Found the x-intercept(s) by setting $y = 0$ and solving the equation (by any of the methods from Unit 1).
c. Found the y-intercept by setting $x = 0$.
d. Found the coordinates of the vertex by writing the equation in completed square form.

1. Sketch the graph of $y = x^2 + 10x + 9$

2. Sketch the graph of $y = x^2 + 6x + 8$

3. Sketch the graph of $y = x^2 - 2x - 3$

4. Sketch the graph of $y = -x^2 - 2x + 8$

Mark your work.

3. Write down the equation of a quadratic given the graph

If necessary watch clip number #257 again.

Or Corbett Maths #265

Practice Exercise 2.3

Complete these questions on file paper.

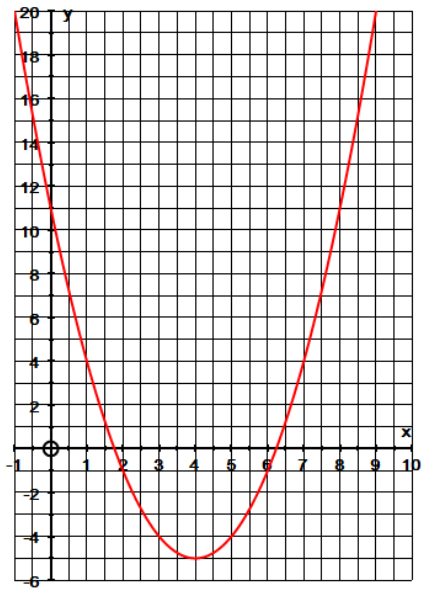
Write down the equations of the graphs shown.

You are now working backwards from what you did before.

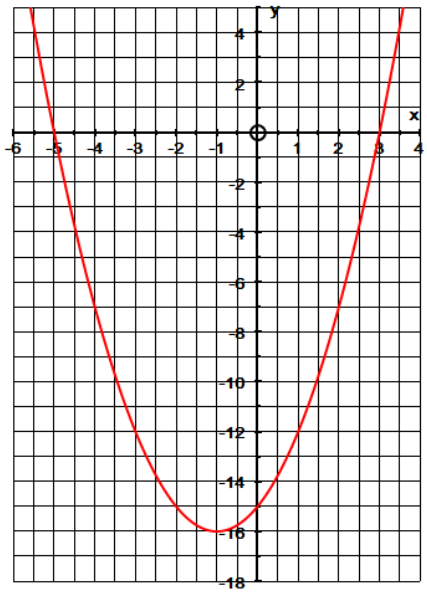
- If you know the coordinates of the vertex you can write the equation in completed square form.
- If you know the x-intercept(s) you can write the equation in factorised form.

In either case check your answer by expanding and simplifying, then checking that the y-intercept is correct.

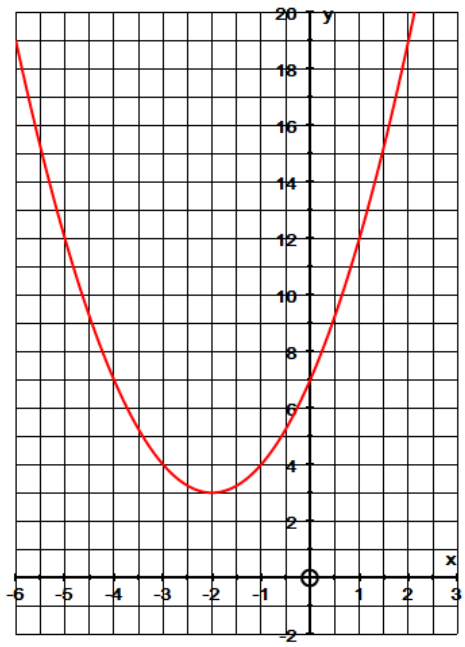
1.



2.



3.



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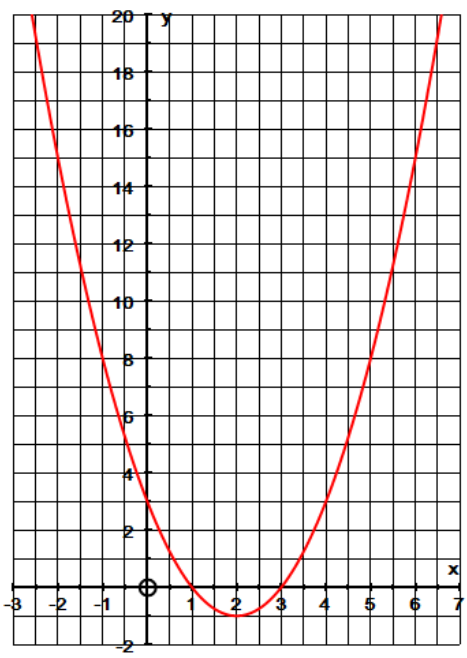

Mark your work.

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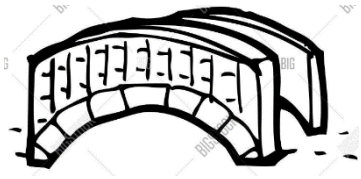


Bridging Unit 3 – Linear Graphs

You need to be able to confidently:

- Write the equation for/draw a straight line using the gradient and intercept
- Find the gradient and y-intercept of a line by rearranging its equation
- Find the equation of a line using gradient and coordinates
- Find the equation of a perpendicular line

1. Write the equation for/draw a straight line using the gradient and intercept

This unit assumes that you can draw a straight line graph by filling in a table of values and plotting points.

Watch clip number #207-209

Or Corbett Maths #187

Practice Exercise 3.1
Complete these questions on squared paper.

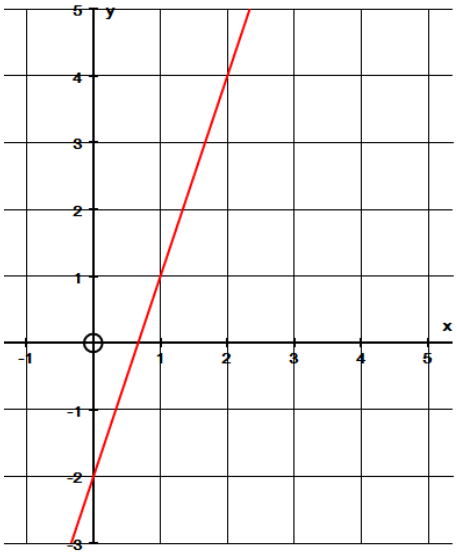
1. Write down the gradient and y-intercept of the line with equation:
   a) $y = 7x - 8$
   b) $y = 2x + 9$
   c) $y = 0.5x$

2. Write down the equation of the line with:
   a) Gradient 5 and y-intercept -2
   b) Gradient 10 and y-intercept 7
   c) Gradient -3 and y-intercept 1

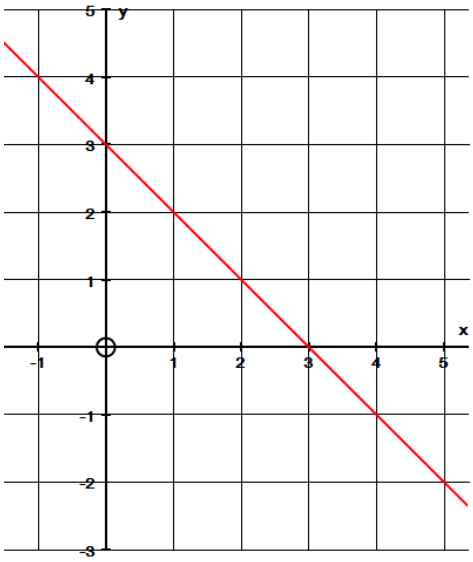
3. On squared paper, draw the graphs from questions 1 and 2.
   Do not use a table of values; use the gradient and y-intercept.

4. Write down the equation of these graphs by identifying the gradient and y-intercept:

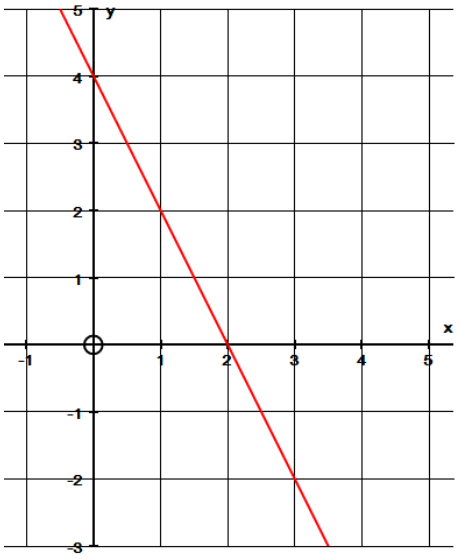
a)



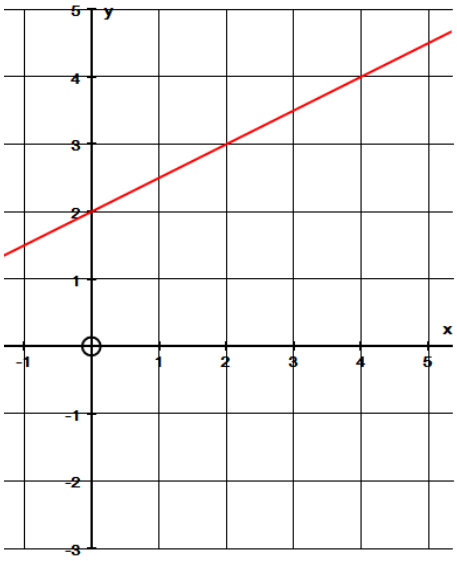
b)



c)



d)



Mark your work.

2. Find the gradient and y-intercept of a line by rearranging its equation

Watch clip number #210

Or Corbett Maths #191

Practice Exercise 3.2
Complete these questions on file paper.

For each question:
a) Use the balance method (showing all your working) to rearrange these equations into the form $y = mx + c$
b) Identify the gradient and y-intercept.

1. $x + y = 13$
2. $\frac{y}{2} = 2x + 3$
3. $2x - y = -7$
4. $4x + 2y = 5$
5. $x = 20 - 4y$
6. $6x = 2y + 3$
7. $6x = \frac{y}{3} + 3$
8. $6x = 7(y - 1)$
9. $6(x - 2) = 5y$
10. $6x - 5y = 4$

Mark your work.

3. Find the equation of a line using gradient and coordinates

Watch clip number #212

Or Corbett Maths #194

Practice Exercise 3.3
Complete these questions on file paper.

Write down the equation of the graph which:

1. Has gradient 3 and goes through (1, 4).
2. Has gradient 7 and goes through (-2, 7).
3. Has gradient -2 and goes through (2, 4).
4. Has gradient -10 and goes through (-3, 5).

5. Goes through (0, 4) and (2, 10).

6. Goes through (2, 10) and (4, 14).

7. Goes through (-1, -4) and (7, 12).

8. Goes through (3, -7) and (11 ,-3).

9. Is parallel to the line $y = 4x + 2$ and goes through the point (4, 5).

10. Is parallel to the line $x + y = 6$ and goes through the point (3, -7).

11. Is parallel to the line $x - y = 5$ and goes through the point (3, 10).

12. Is parallel to the line $x - 2y = 16$ and goes through the point (-4, 5).

Mark your work.

4. Find the equation of a perpendicular line

Watch clip number #216

Or Corbett Maths #197

Practice Exercise 3.4
Complete these questions on file paper.

1. Find the equation of the line which is perpendicular to the line $y = 2x - 8$ and goes through the point (0, 4).

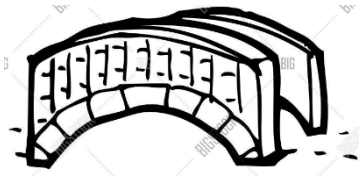
2. Find the equation of the line which is perpendicular to the line $y = -3x + 13$ and goes through the point (1, 2).

3. Find the equation of the line which is perpendicular to the line $y = \frac{1}{2}x$ and goes through the point (5, 8).

4. Find the equation of the line which is perpendicular to the line $y = \frac{1}{8}x - 3$ and goes through the point (-1, 4).

5. Find the equation of the line which is perpendicular to the line $y = -\frac{2}{3}x + 23$ and goes through the point (3, -5).

Mark your work.



Bridging Unit 4 – Indices and Surds

You need to be able to confidently:

- Apply the Index Laws with numbers or algebra
- Work with negative and fractional indices
- Manipulate expressions containing surds (including simplifying surds, expanding brackets containing surds and rationalising a denominator)

1. Apply the Index Laws with numbers or algebra

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Watch clip number #110

Or Corbett Maths #174

Practice Exercise 4.1a

Complete these questions on file paper. **Do not use a calculator.**

1 Write as a power of 2

a $2^4 \times 2^5$ **b** $2^3 \times 2^4$ **c** $2^2 \times 2^6$ **d** $2^4 \times 2^3$ **e** $2^4 \times 2^6$

2 Write as a power of 3

a $3^4 \div 3^2$ **b** $3^5 \div 3^2$ **c** $3^4 \div 3$ **d** $3^6 \div 3^2$ **e** $3^{10} \div 3^4$

3 Write as a power of a single number

a $4^4 \div 4^2$ **b** $5^7 \div 5^2$ **c** $3^4 \times 3^2$ **d** $6^4 \times 6^3$ **e** $10^4 \div 10^2$

4 Find the value of n

a $3^n \div 3^2 = 3^3$ **b** $8^5 \div 8^n = 8^2$ **c** $2^5 \times 2^n = 2^{10}$ **d** $3^n \times 3^5 = 3^9$ **e** $2^6 \times 2^3 = 2^n$

5 Work out

a $3^4 \div 3^2$ **b** $4^5 \div 4^3$ **c** $2^5 \div 2^2$ **d** $10^4 \times 10^2$ **e** $6^5 \div 6^5$

6 Write as a power of 3

a $\dfrac{3^3 \times 3^5}{3^4}$ **b** $(3^3)^2$ **c** $\dfrac{3 \times 3^7}{3^4}$ **d** $\dfrac{3^9}{3^4 \times 3^3}$ **e** $\dfrac{3^2 \times 3^{10}}{3^2 \times 3^5}$

7 Write as a power of a single number

a $\dfrac{2^3 \times 2^4}{2^5}$ **b** $\dfrac{3^4 \times 3^3}{3^4}$ **c** $\dfrac{5^3 \times 5^5}{5^6}$ **d** $\dfrac{10^8 \times 10^3}{10^7}$ **e** $\dfrac{4^5 \times 4}{4^2}$

8 Work out

a $\dfrac{5^5}{5^2 \times 5^2}$ **b** $\dfrac{3^4}{3^2 \times 3^2}$ **c** $\dfrac{4^7}{4^2 \times 4^3}$ **d** $\dfrac{2^3 \times 2^4}{2^4 \times 2^2}$ **e** $\dfrac{3 \times 3^7}{3^4 \times 3^2}$

9 Work out the value of n in the following.

a $40 = 5 \times 2^n$ **b** $32 = 2^n$ **c** $50 = 5^n \times 2$ **d** $48 = 3 \times 2^n$ **e** $54 = 2 \times 3^n$

Mark your work.

Practice Exercise 4.1b

Complete these questions on file paper.

1 Simplify

a $x^3 \times x^2$ **b** $y^5 \times y^3$ **c** $n \times n^6$ **d** $q^7 \times q$ **e** $x^5 \div x^3$

f $y^7 \div y^3$ **g** $p^5 \div p^4$ **h** $q^7 \div q$ **i** $y \times y^4 \times y^3$ **j** $q^4 \times q \div q^3$

2 Simplify

a $3x^2 \times x^5$ **b** $4p \times 2p^4$ **c** $4p \times 5p$ **d** $2 \times 2r^8 \times 4r$

e $6y^6 \div 2y^3$ **f** $12q^2 \div 6q$ **g** $8x^9 \div 2x^8$ **h** $4q \div 2q$

i $2y^2 \times 3y^3 \times y^3$ **j** $6q \times 5q^4 \div 2q^5$

3 Simplify

a **i** $x^2 \times x$ **ii** $x^5 \div x^2$ **iii** $(x^2 \times x) + (x^5 \div x^2)$

b $(8y^6 \div 2y^2) - (2y^2 \times y^2)$

4 Simplify

a $3a^4 \times a^3b^2$ **b** $2ab^4 \times 4a^3b$ **c** $5p^4q^3 \times 2q^3p^2$

d $18x^8y^6 \div 6x^3y^2$ **e** $12a^3b^5 \div 3a^3b$ **f** $20p^4q \div 2p^3q^2$

5 Find the value of

a $4x^0$ **b** $(xy)^0$

6 Write as a power of x

a $\dfrac{1}{x^4}$ **b** $\dfrac{1}{x}$ **c** $\dfrac{1}{x^4 \times x^3}$ **d** $\dfrac{1}{x^4 \div x}$ **e** $\dfrac{1}{x^5 \div x^7}$

7 Simplify

a $(x^5)^3$ **b** $(2y^2)^4$ **c** $(a^2b^4)^5$ **d** $(3a^3b)^3$

e $(x^2)^{-1}$ **f** $(4y^{-2})^2$ **g** $(a^{-4})^{-3}$ **h** $(-2b^{-4})^{-2} \div b^2$

8 Simplify

a $(x^2)^0$ **b** $(y^0)^4$ **c** $(2a^3b^{-2})^{-3} \times (2a^{-3}b)^3$

Mark your work.

2. Work with negative and fractional indices

If necessary watch clip number #110 again

Or Corbett Maths #173 & #175

Practice Exercise 4.2a

Complete these questions on file paper. **Do not use a calculator.**

1 Work out the value of the following.

a 2^{-1} **b** 3^{-2} **c** 5^{-1} **d** 10^{-3} **e** 2^{0}

f 2.5^{-1} **g** $\left(\frac{1}{3}\right)^{-1}$ **h** $\left(\frac{2}{3}\right)^{-2}$

2 Simplify the following.

a $3^2 \times 3^{-3}$ **b** $4^{-2} \times 4$ **c** $5^4 \times 5^{-2}$ **d** $6^2 \times 6^{-4}$ **e** $2^2 \times 2^{-5}$

3 Simplify the following.

a $4^{-2} \div 4^{-1}$ **b** $3^2 \div 3^{-1}$ **c** $2^{-2} \div 2^{-4}$ **d** $10^{-4} \div 10^{-3}$ **e** $5^{-3} \div 5^{-1}$

4 Simplify the following.

a $\frac{2^4 \times 2^2}{2^7}$ **b** $\frac{3^4 \times 3^{-2}}{3^3}$ **c** $\frac{5^{-2} \times 5^2}{5}$ **d** $\frac{4^{-3} \times 4^3}{4^{-2}}$ **e** $\frac{2^{-4} \times 2^2}{2^{-7}}$

5 Simplify the following.

a $\frac{2^4}{2^7 \times 2^{-2}}$ **b** $\frac{3^4 \times 3^2}{3 \times 3^7}$ **c** $\frac{5^4 \times 5^{-2}}{5^2 \times 5^{-1}}$ **d** $\frac{4^4 \times 4^{-2}}{4^{-1}}$ **e** $\frac{2^4 \times 2^2}{2^7 \times 2^{-1}}$

6 Find the value of n in each of the following.

a $2^n = \frac{2^2}{2^5}$ **b** $3 \times 3^n = \frac{3^3}{3^5}$ **c** $\frac{5^n}{5} = \frac{5^2}{5^5}$ **d** $4^2 \times 4^n = \frac{4^2}{4^6}$

Mark your work.

Practice Exercise 4.2b

Complete these questions on file paper. **Do not use a calculator.**

1 Work out the value of

a $9^{\frac{1}{2}}$ **b** $25^{\frac{1}{2}}$ **c** $100^{\frac{1}{2}}$ **d** $4^{\frac{1}{2}}$ **e** $\left(\frac{1}{4}\right)^{\frac{1}{2}}$

2 Work out the value of

a $27^{\frac{1}{3}}$ **b** $1000^{\frac{1}{3}}$ **c** $-64^{\frac{1}{3}}$ **d** $125^{\frac{1}{3}}$ **e** $\left(\frac{1}{125}\right)^{\frac{1}{3}}$

3 Work out as a single fraction the value of

a $\left(\frac{1}{2}\right)^4$ **b** $\left(\frac{1}{3}\right)^2$ **c** $\left(\frac{2}{3}\right)^2$ **d** $\left(\frac{2}{5}\right)^2$ **e** $\left(\frac{3}{4}\right)^3$

4 Work out the value of

a $27^{\frac{2}{3}}$ **b** $1000^{\frac{2}{3}}$ **c** $64^{\frac{2}{3}}$ **d** $16^{\frac{3}{4}}$ **e** $25^{\frac{3}{2}}$

5 Work out as a single fraction the value of

a $25^{-\frac{1}{2}}$ **b** $9^{-\frac{1}{2}}$ **c** $27^{-\frac{1}{3}}$ **d** $8^{-\frac{2}{3}}$ **e** $64^{-\frac{3}{2}}$

6 Find the value of n.

a $\frac{1}{\sqrt{5}} = 5^n$ **b** $(\sqrt{7})^5 = 7^n$ **c** $(\sqrt[3]{2})^{11} = 2^n$

Mark your work.

3. Manipulate expressions containing surds

(including simplifying surds, expanding brackets containing surds and rationalising a denominator)

Watch clip numbers #115, #117-119

Or Corbett Maths #308 & #307

Practice Exercise 4.3

Complete these questions on file paper.

1 Find the value of the integer k.

a $\sqrt{8} = k\sqrt{2}$ **b** $\sqrt{18} = k\sqrt{2}$ **c** $\sqrt{50} = k\sqrt{2}$ **d** $\sqrt{80} = k\sqrt{5}$ **e** $\sqrt{72} = k\sqrt{2}$

2 Expand these expressions. Write your answers in the form $a + b\sqrt{c}$ where a, b and c are integers.

a $\sqrt{3}(2 + \sqrt{3})$ **b** $(\sqrt{3} + 1)(2 + \sqrt{3})$ **c** $(\sqrt{5} - 1)(2 + \sqrt{5})$

d $(\sqrt{7} + 1)(2 - 2\sqrt{7})$ **e** $(2 - \sqrt{3})^2$

3 Rationalise the denominators.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{2}{\sqrt{7}}$ **d** $\frac{3}{\sqrt{2}}$ **e** $\frac{5}{\sqrt{11}}$

4 Rationalise the denominators and simplify your answers.

a $\frac{2}{\sqrt{6}}$ **b** $\frac{3}{\sqrt{12}}$ **c** $\frac{5}{\sqrt{10}}$ **d** $\frac{2}{\sqrt{2}}$ **e** $\frac{10}{\sqrt{5}}$

5 Rationalise the denominators and give your answers in the form $a + b\sqrt{c}$ where a, b and c are integers.

a $\frac{2 + \sqrt{2}}{\sqrt{2}}$ **b** $\frac{2 - \sqrt{2}}{\sqrt{2}}$ **c** $\frac{10 + \sqrt{5}}{\sqrt{5}}$ **d** $\frac{5 - \sqrt{5}}{\sqrt{5}}$ **e** $\frac{14 + \sqrt{7}}{\sqrt{7}}$

6 The lengths of the two shorter sides of a right-angled triangle are $\sqrt{7}$ cm and 3 cm. Find the length of the hypotenuse.

7 The length of the side of a square is $(1 + \sqrt{2})$ cm. Work out the area of the square. Give your answer in the form $(a + b\sqrt{2})$ cm^2 where a and b are integers.

8 The length of a rectangle is $(3 + \sqrt{5})$ cm. The width of the rectangle is $(4 - \sqrt{5})$ cm.

Work out **a** the perimeter of the rectangle **b** the area of the rectangle.

Mark your work.

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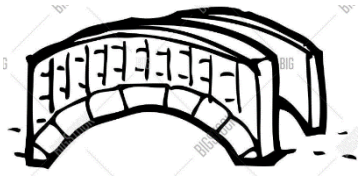


Bridging Unit 5 – Simultaneous Equations

You need to be able to confidently:

- Solve linear simultaneous equations by elimination
- Solve linear simultaneous equations by substitution
- Solve simultaneous equations where one is linear and one quadratic or a circle
- Solve simultaneous equations graphically

1. Solve linear simultaneous equations by elimination

hegartymaths

Watch clip number #192-193

Or Corbett Maths #295

Practice Exercise 5.1

Complete these questions on file paper.

1. (a) $6x + y = 18$, $4x + y = 14$ (b) $4x + 2y = 10$, $x + 2y = 7$ (c) $9x - 4y = 19$, $4x + 4y = 20$

   (d) $2x + y = 36$, $x - y = 9$ (e) $6x - 3y = 12$, $4x - 3y = 2$ (f) $3x - 6y = 6$, $2x - 6y = 3$

2. (a) $3x + 2y = 23$, $2x - y = 6$ (b) $3x - 3y = 9$, $2x + y = 12$ (c) $4x + 2y = 34$, $3x + y = 21$

   (d) $9x - 4y = 59$, $2x - y = 12$ (e) $2x + 8y = 43$, $x + 3y = 18$ (f) $6x + 3y = 45$, $2x - 2y = 12$

3. (a) $2x + 2y = 14$, $5x - 3y = 19$ (b) $2x + 3y = 1$, $7x + 2y = -22$ (c) $5x + 3y = 22$, $2x + 4y = 20$

   (d) $5x - 6y = 28$, $4x - 4y = 24$ (e) $3x + 2y = 7$, $2x + 9y = 43$ (f) $3x + 3y = -6$, $4x - 4y = -24$

Mark your work.

2. Solve linear simultaneous equations by substitution

Watch clip number #194.

Or Corbett Maths #296 & #298

Practice Exercise 5.2

Complete these questions on file paper.

1. (a) $y = x + 3$, $y = x^2 + 5x - 2$ (b) $y = x^2 + x - 14$, $y = x - 5$ (c) $y = 2x - 1$, $y = x^2 - 2x + 2$

   (d) $y = 2x^2 + 9x + 1$, $y = 3x + 9$ (e) $y = 2x^2 + x + 1$, $y = x^2 - 5x - 7$ (f) $y = -x^2 + 5x + 2$, $y = 3x^2 - x - 2$

2. (a) $x + y = 4$, $y = x^2 + 3x - 1$ (b) $x + y = 7$, $xy = 10$ (c) $x^2 + y^2 = 13$, $x + y = 5$

   (d) $2x - y + 4 = 0$, $y = x^2 + x - 2$ (e) $x^2 + y^2 = 29$, $7 + x + y = 0$ (f) $xy = -6$, $x + 2y = -4$

3. (a) $2x + y = 7$, $x^2 - y^2 = 8$ (b) $x^2 + y^2 = 20$, $y = x + 3$ (c) $y = x^2 - 9x - 3$, $y = x$

   (d) $2x^2 + y^2 = 10$, $2x - y = 5$ (e) $y = x^2 + x - 7$, $4x + 2y + 1 = 0$ (f) $y = x - 2$, $2x^2 - xy = 11$

Mark your work.

3. Solve simultaneous equations where one is linear and one quadratic or a circle

Watch clip number #246

Or Corbett Maths #298

Practice Exercise 5.3
Complete these questions on file paper.

1 Solve these simultaneous equations.

a $y = 2x$ and $y = 2x^2$

b $y = x + 3$ and $y = x^2 + 3x$

c $y = 3x - 1$ and $y = x^2 - 5$

d $y = 6 - x^2$ and $y = 4x + 1$

2 Solve

a $y - 3x = 4$ and $y = 2x^2 - 5$

b $x + y = 2$ and $y = 3x^2 - 2$

c $y + 2x = 5$ and $y = 2x^2 + x$

d $x - y = 3$ and $y = x^2 - 2x - 1$

e $2x - y = -8$ and $y = x^2$

f $2x + 3y = 13$ and $y = x^2 - 1$

Mark your work.

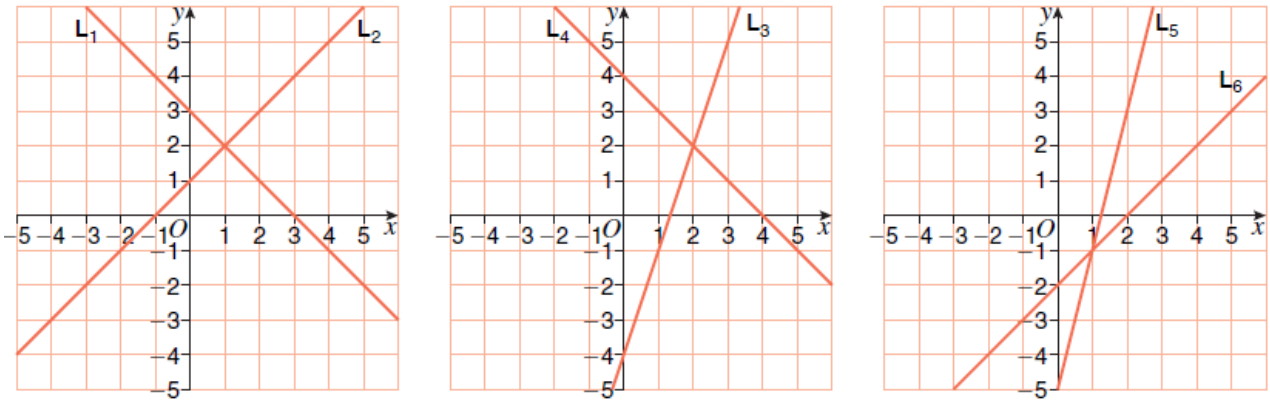
4. Solve simultaneous equations graphically

Watch clip number #218-219 & #259

Or Corbett Maths #297

Practice Exercise 5.4
Complete these questions on **squared paper**.

1. The diagrams show six lines labelled L_1, L_2, L_3, L_4, L_5 and L_6.



Use the diagrams to solve these simultaneous equations.

i $x + y = 3$
$y = x + 1$

ii $x + y = 4$
$y = 3x - 4$

iii $y = x - 2$
$y = 4x - 5$

Solve these simultaneous equations by drawing both graphs on one set of axes (from -5 to 5) and finding the point of intersection:

2 $x + y = 0$
$y = x + 2$

3 $y = x - 2$
$y = 2x$

4 $x + y = 1$
$y = -2x - 1$

5 $y = x + 1$
$y = 2x + 4$

6 $y = 2x - 5$
$x + y = 4$

Solve these simultaneous equations by drawing both graphs on one set of axes and finding the points of intersection, remembering to pair up the values correctly:

7. **a** $y = 3$ and $y = x^2 + 2x$
b $y = 5$ and $y = x^2 - 4x$
c $y = -4$ and $y = x^2 - 5x$
d $y = -1$ and $y = 2x^2 + 5x + 1$

8. **a** $y = x + 6$ and $y = x^2$
b $y = x$ and $y = x^2 - 2$
c $y = x + 1$ and $y = 2x^2$
d $y = 4 - x$ and $y = 2x^2 + 3$
e $y = x$ and $y = x^2 + 7x + 5$
f $x + 2y = 0$ and $y = 2x^2 - 4x - 1$

Mark your work.

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Bridging Unit 6 – Rearranging Equations

You need to be able to confidently:

- Change the subject of a formula

1. Change the subject of a formula

Watch clip number #281-286

Or Corbett Maths #7-8

Practice Exercise 6.1

Complete these questions on file paper.
Solve each of the inequalities below:

1. (a) $4x + c = w$ (b) $dx - t = 8$ (c) $x^2 + 3 = h$

   (d) $2x + 2y = P$ (e) $s = x^2 - 3$ (f) $y = xz + s$

   (g) $\frac{x}{n} + 2 = w$ (h) $\frac{x}{6} - 5 = w$ (i) $\frac{x + 3}{c} = h$

   (j) $3y = 4x + 1$ (k) $x^2 + a = v$ (l) $x^3 - 4 = 5y$

2. (a) $A = \frac{1}{2}(x + y)$ (b) $A = \pi r^2 + 2\pi rx$

   (c) $T = 3x^2 - y$ (d) $s = \frac{m}{ax}$

   (e) $s = uy + \frac{1}{2}xy^2$ (f) $\frac{1}{3}w = \frac{1}{4}x + t$

   (g) $j = \frac{x + 3}{d}$ (h) $g = \frac{t}{x - 2}$

   (i) $p = 3(y + 2x)^2$ (j) $12w = \frac{3}{4}(2x + a)$

3.

(a) $5(m + y) = 4(m - 3y)$

(b) $3(3m + 4) = 7(m + 2a)$

(c) $15(2m + 3c) = 5(m + 7c)$

(d) $9m + 4c = 2(a + 3m)$

(e) $a(c + m) = 2(c + 3m)$

(f) $w(m + n) = x(m - n)$

(g) $8 = \dfrac{m + 3c}{m - f}$

(h) $y = \dfrac{m + 4}{m + 5}$

(i) $y = 3mt - a^2m$

(j) $r(c + 7) = 3m + 5$

Mark your work.

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Bridging Unit 7 – Inequalities

You need to be able to confidently:

- Solve linear inequalities
- Solve quadratic inequalities
- Plot a region using inequalities on a graph

1. Solve linear inequalities

Watch clip number #169-172

Or Corbett Maths #178-179

Practice Exercise 7.1

Complete these questions on file paper.
Solve each of the inequalities below:

1. (a) $2x+1 \leq 9$ (b) $3x-5 > 16$ (c) $4x+8 < 32$ (d) $5x-2 \geq 68$

   (e) $\frac{x}{2}+1 \leq 5$ (f) $\frac{x}{9}-6 > 4$ (g) $\frac{x+3}{2} \geq 5$ (h) $\frac{x-5}{4} > 2$

2. (a) $5(x-3) \geq 40$ (b) $6(x+2) < 42$ (c) $2(5x+1) \leq 36$

   (d) $4(x-2) < 18$ (e) $2(2x-9) \geq 22$ (f) $3(2x+7) \leq 9$

3. (a) $4x+3 > 2x+11$ (b) $x+1 \geq 3x-18$

   (c) $13x-12 < 3x+13$ (d) $7x-5 \geq 3x+11$

4. (a) $6 < x+3 < 10$ (b) $4 \leq 2x \leq 7$ (c) $1 \leq 3x < 9$

   (d) $4 < \frac{x}{5} < 6$ (e) $9 \leq 2x+3 \leq 25$ (f) $-3 \leq \frac{x}{4}-1 < 0$

Mark your work.

2. Solve quadratic inequalities

Watch clip number #277

Or Corbett Maths #378

Practice Exercise 7.2
Complete these questions on file paper.
Solve the following inequalities:

1. (a) $(x-4)(x-1)<0$ (b) $(x-2)(x+1)<0$ (c) $(x+7)(x+3)\leq 0$
   (d) $(x-5)(x+4)\leq 0$ (e) $x(x-9)>0$ (f) $(x+6)(x-5)>0$
   (g) $(x+10)(x+1)\geq 0$ (h) $(x-7)(x+7)\geq 0$ (i) $(x+8)(x+2)<0$

2. (a) $x^2+5x+6>0$ (b) $x^2+7x+10<0$ (c) $x^2-4x-5\leq 0$
   (d) $x^2+2x-24>0$ (e) $x^2-6x+8\geq 0$ (f) $x^2+3x-4<0$
   (g) $x^2-x-56>0$ (h) $x^2+9x+18<0$ (i) $x^2-13x+22\leq 0$

3. (a) $x^2-2x<15$ (b) $x^2+6x>x-4$ (c) $x^2<36$
   (d) $x^2>121$ (e) $2x^2-x-12\leq x^2-2x$ (f) $6x>x^2-8x+40$

4. (a) $5x^2+7x+2>0$ (b) $3x^2+8x-3<0$ (c) $2x^2-9x+4>0$
   (d) $4x^2-3x-1\geq 0$ (e) $6x^2-13x+7<0$ (f) $2x^2+x-6\leq 0$
   (g) $4x^2-11x+6>0$ (h) $4x^2-27x+18>0$ (i) $15x^2+4x-35<0$

Mark your work.

3. Plot a region using inequalities on a graph

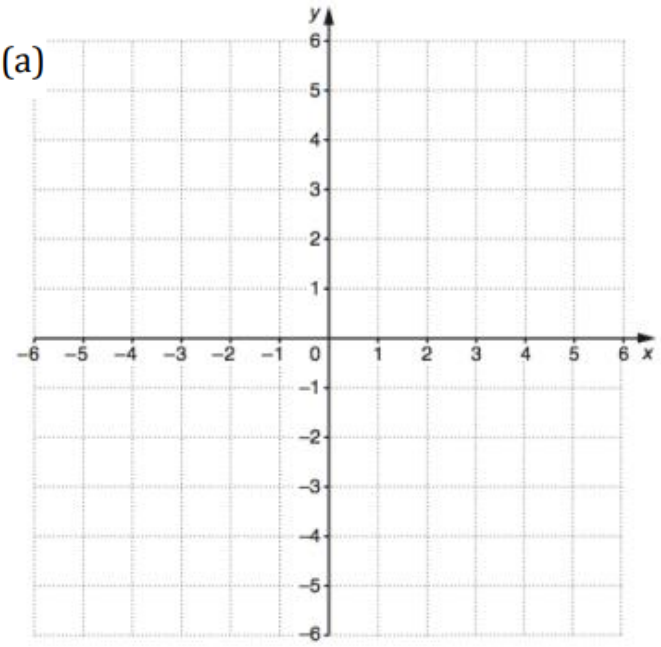
Watch clip number #273-276

Or Corbett Maths #182

Practice Exercise 7.3
Print the following pages or draw your own axes to complete these questions.
Shade the region that satisfies the inequality/inequalities:

1. (a) $y<x+1$ (b) $y\leq 2x+2$ (c) $y>3x-1$
   (d) $y\geq x+3$ (e) $y>2x$ (f) $y\leq 4x$

(a)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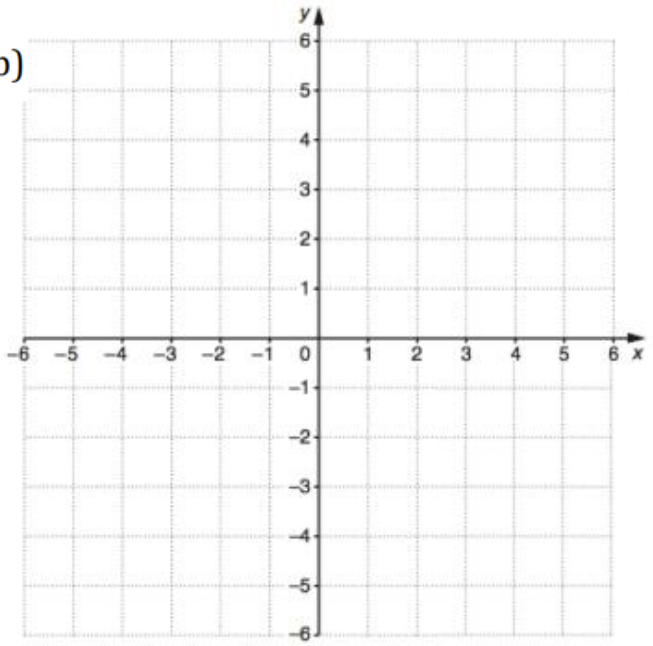
(b)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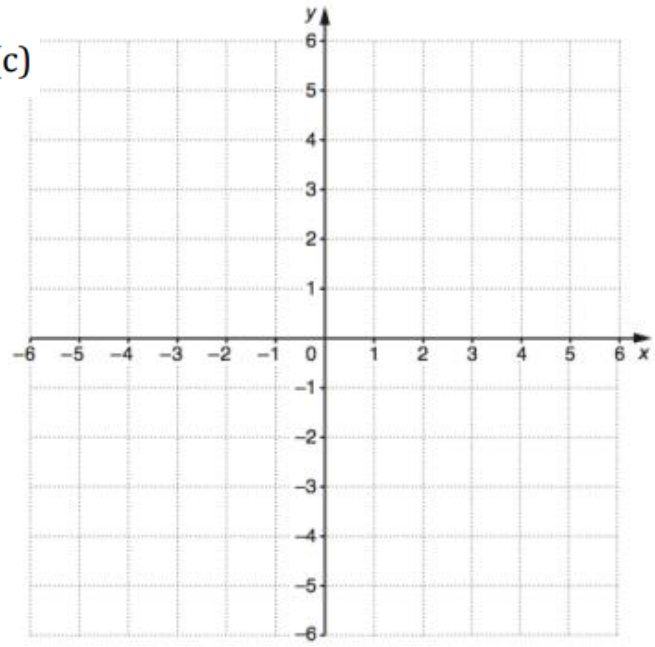
(c)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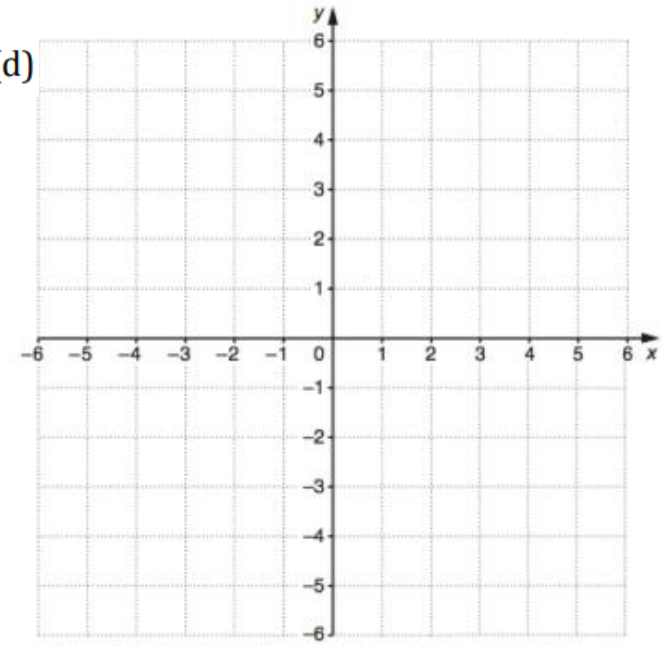
(d)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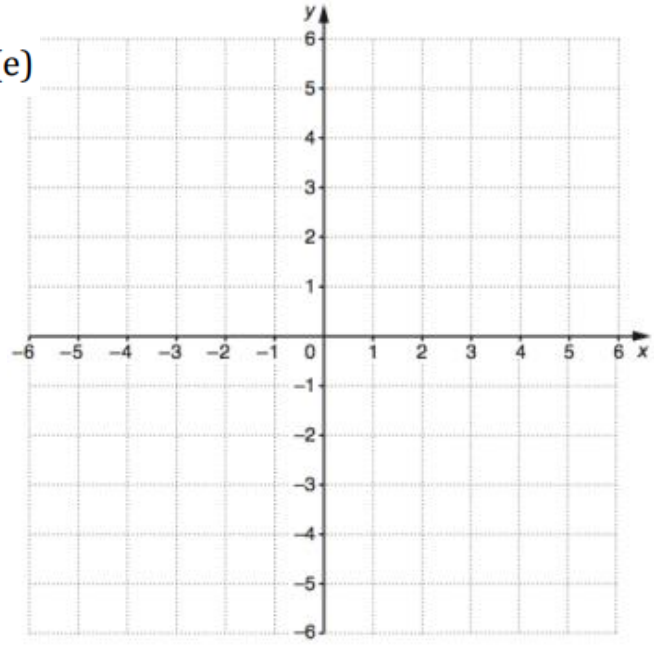
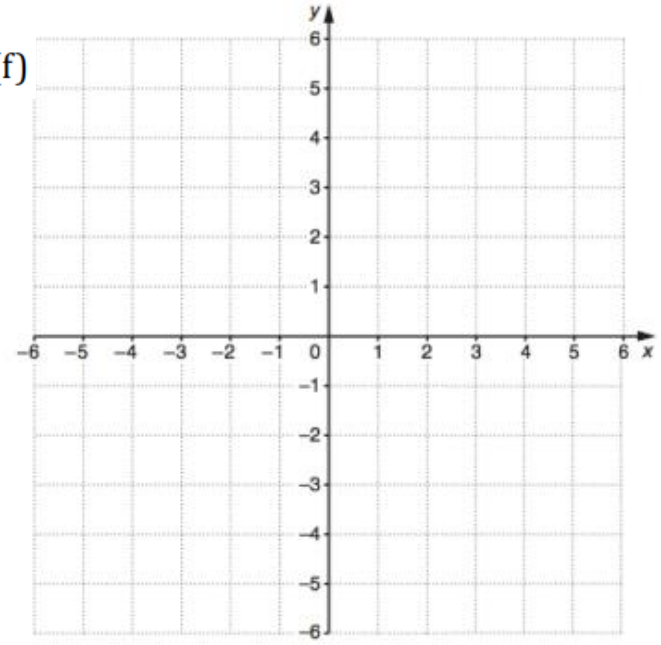
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

(e)


y
x
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

(f)


y
x
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

2.

(a) $y > x - 1$, $x \geq -2$ and $y < 2$

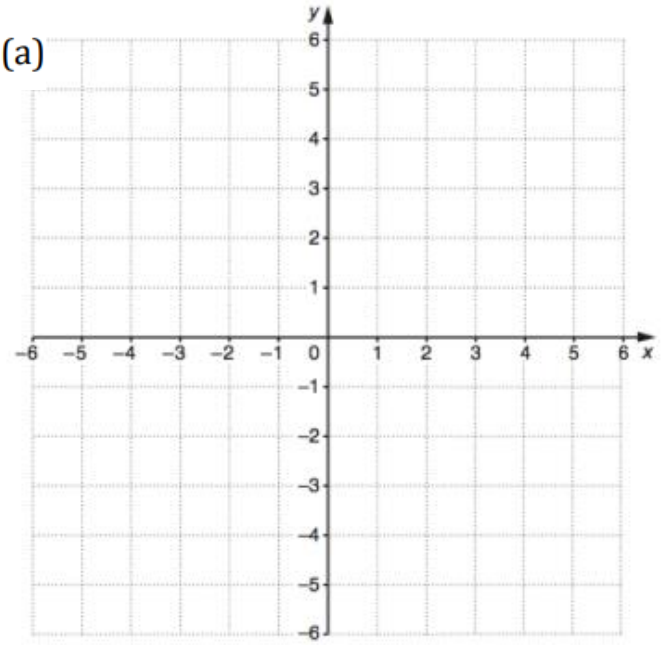
(b) $y \leq 2x$, $x \leq 2$ and $y > -4$

(c) $y \leq -2x + 2$, $x \geq 0$ and $y > x - 4$

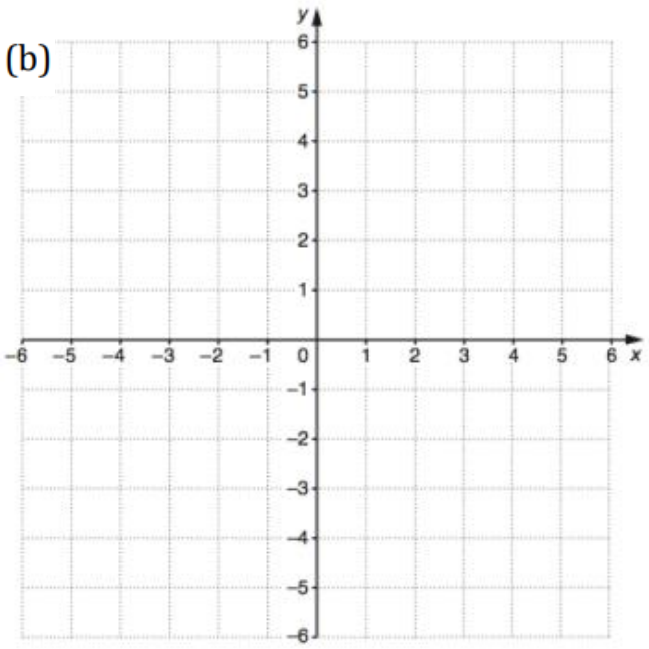
(d) $x + y < 3$, $-2 \leq x < 3$ and $y \geq 0$

(e) $y \leq 5x - 4$, $y > x - 4$ and $y \leq -\frac{1}{2}x + 2$

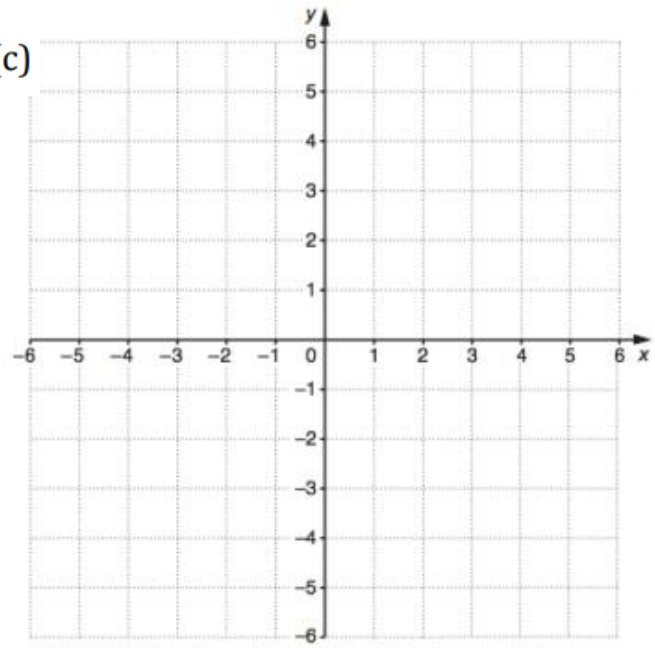
(a)



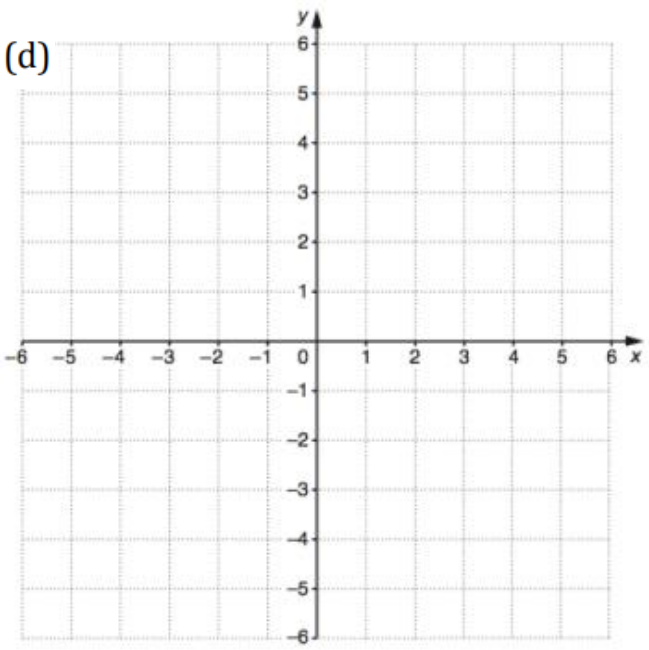
(b)



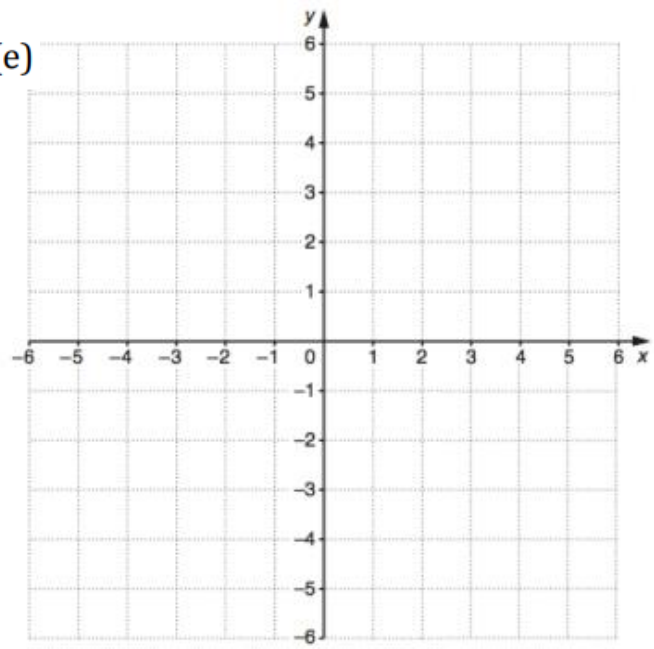
(c)



(d)

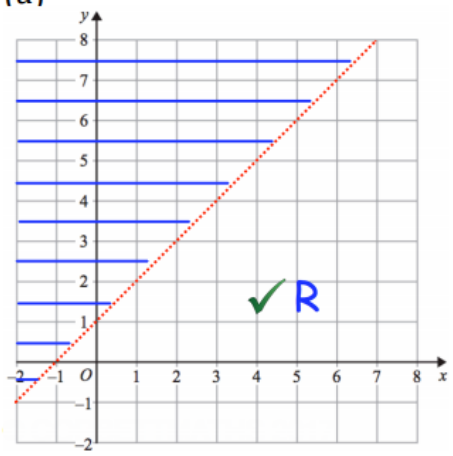


(e)

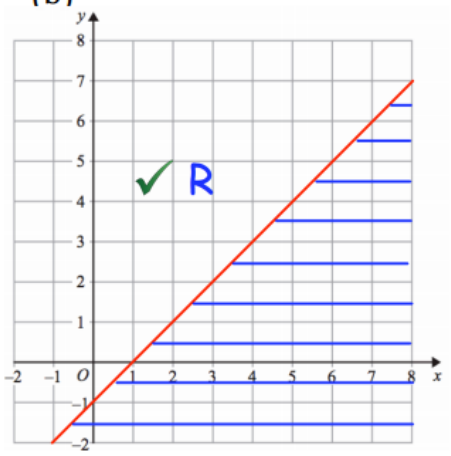


3. Write down the inequality represented by an 'R' in each diagram below:

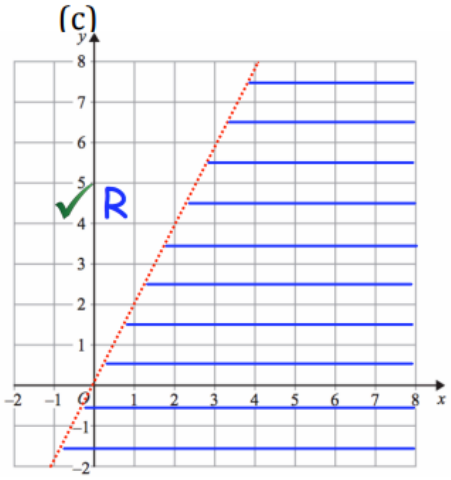
(a)



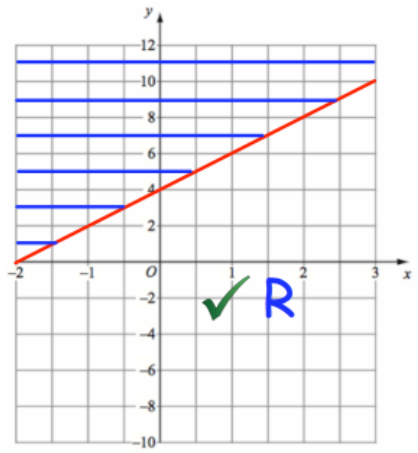
(b)



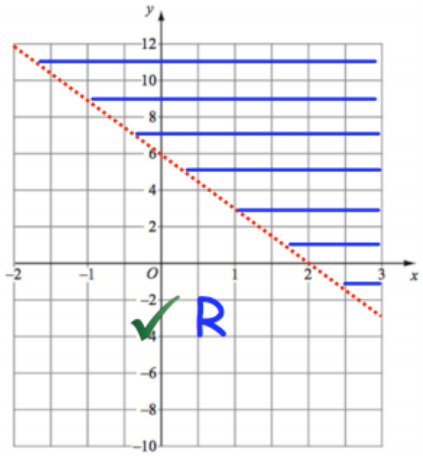
(c)



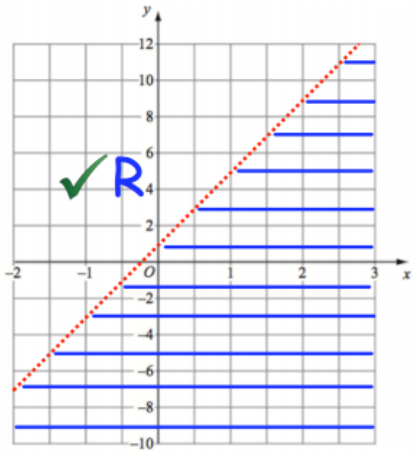
(d)



(e)

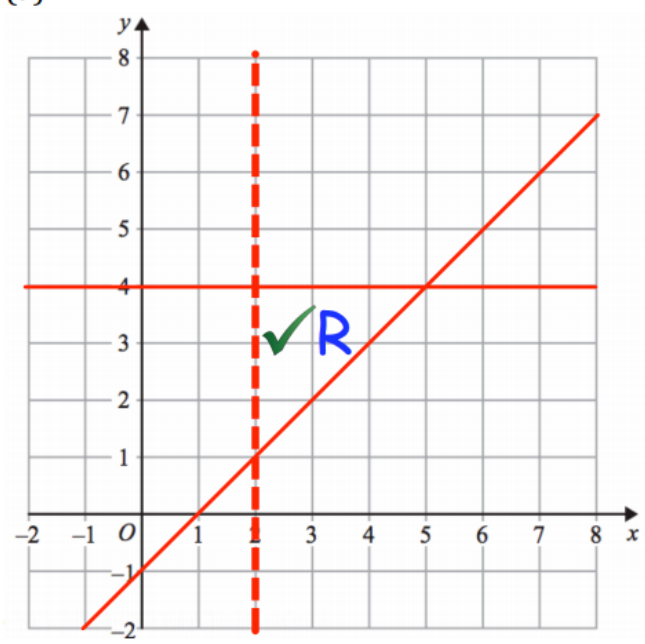


(f)

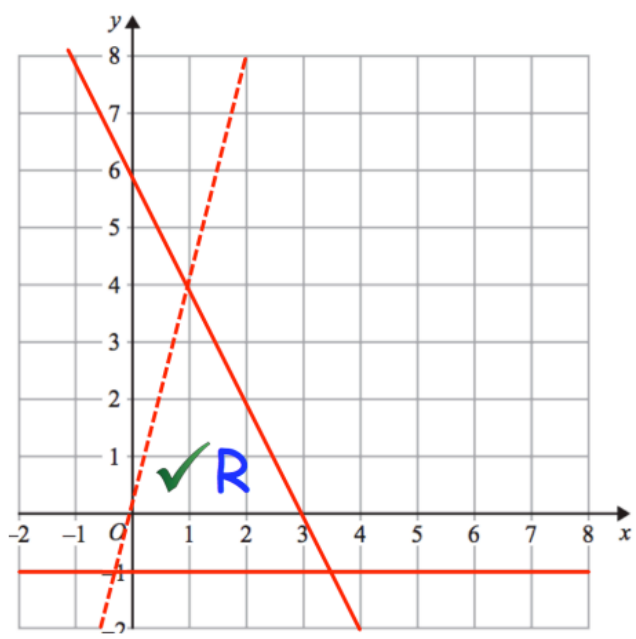


4. State the inequalities that the region 'R' satisfies:

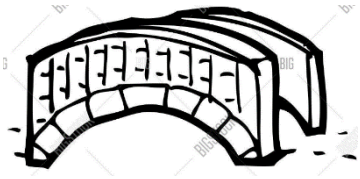
(a)



(b)



Mark your work.



Bridging Unit 8 – Trigonometry

You need to be able to confidently:

- Find sides and angles in triangles using SOHCAHTOA in 2d and 3d
- Find angles, sides and areas in triangles using the Sine, Cosine and Area rules

1. Find sides and angles in triangles using SOHCAHTOA in 2d and 3d

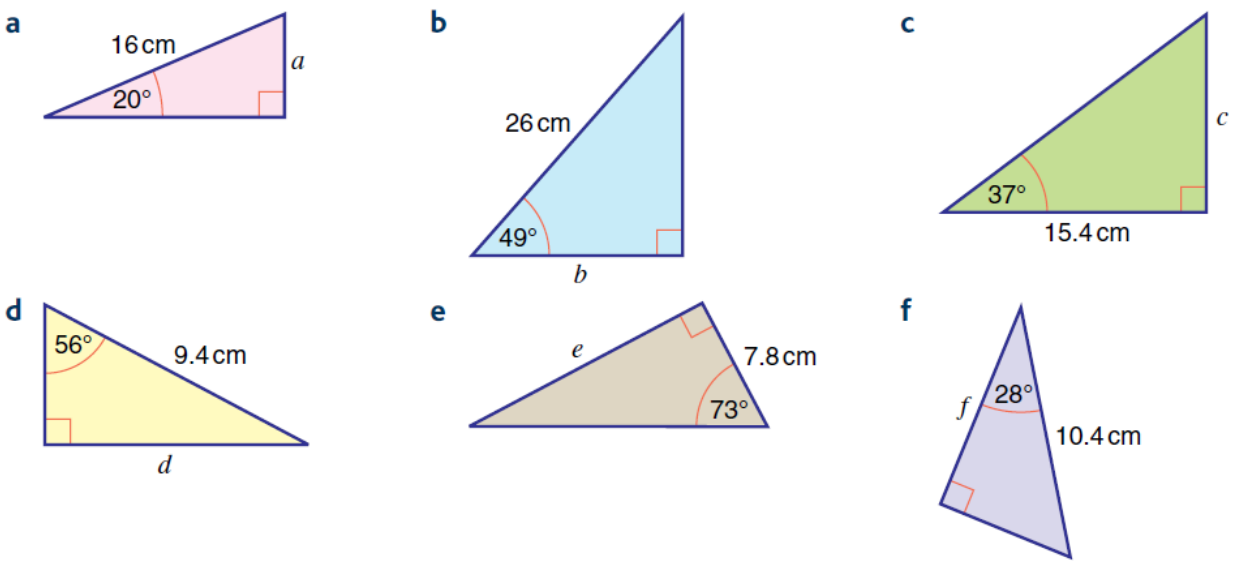
Watch clip number #509 & #511

Or Corbett Maths #330-332

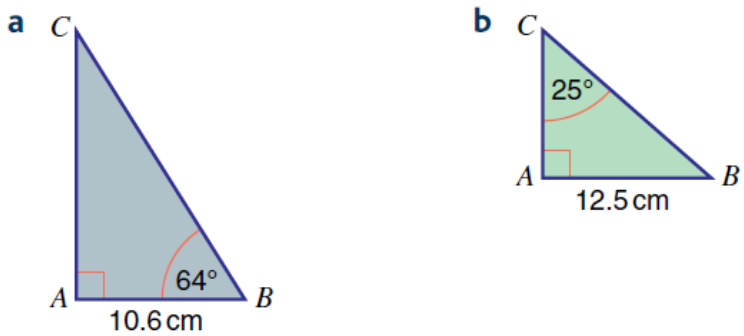
Practice Exercise 8.1

Complete these questions on file paper.

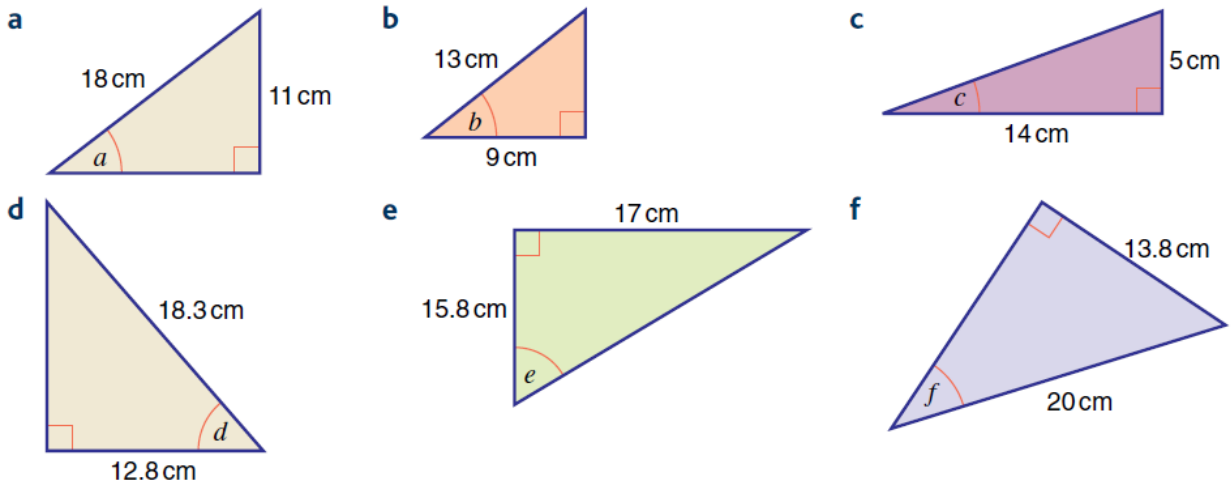
1. Find the lengths of the lettered sides in these triangles:



2. Find the length of BC in these triangles:

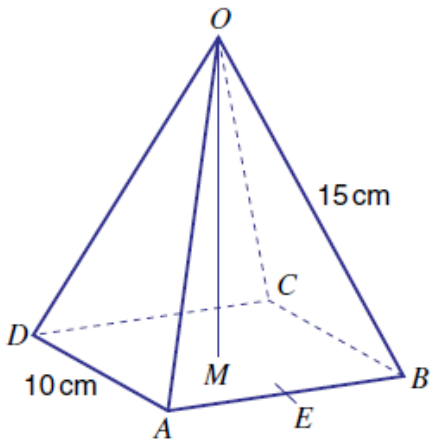


3. Find the marked angle in these triangles:



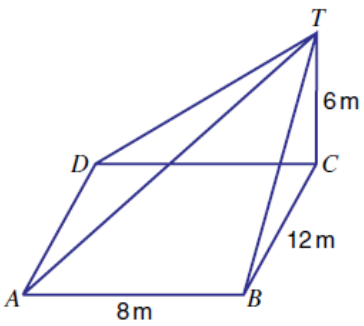
4. The diagram shows a square-based pyramid.
The lengths of sides of the square base, $ABCD$, are 10 cm and the base is on a horizontal plane.
The centre of the base is the point M and the vertex of the pyramid is O, so that OM is vertical.
The point E is the midpoint of the side AB.
$OA = OB = OC = OD = 15$ cm.

a Calculate the length of **i** AC **ii** AM.
b Calculate the length of OM.
c Calculate the size of angle OAM.
d Hence find the size of angle AOC.
e Calculate the length of OE.
f Calculate the size of angle OAB.



5. $ABCD$ is a horizontal rectangular lawn in a garden and TC is a vertical pole. Ropes run from the top of the pole, T, to the corners, A, B and D, of the lawn.

a Calculate the length of the rope TA.
b Calculate the size of the angle made with the lawn by
i the rope TB **ii** the rope TD **iii** the rope TA.



Mark your work.

2. Find angles, sides and areas in triangles using the Sine, Cosine and Area rules

Watch clip number #516, #521, #523, #527, #529

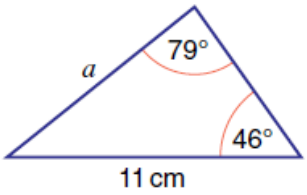
Or Corbett Maths #333-337

Practice Exercise 8.2

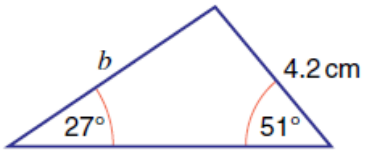
Complete these questions on file paper.

1. Find the lengths of the sides marked with letters in these triangles.

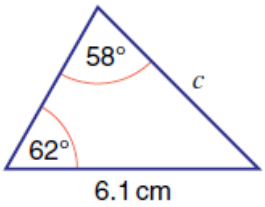
a



b

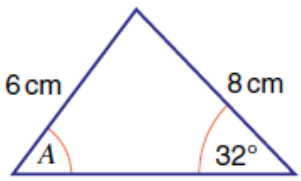


c

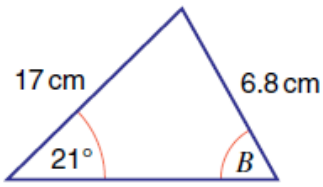


2. Calculate the size of each of the *acute* angles marked with a letter.

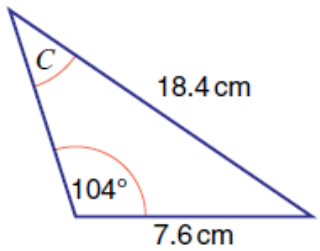
a



b

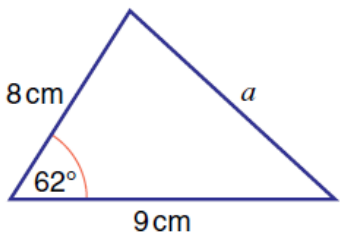


c

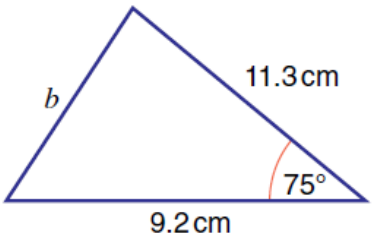


3. Calculate the length of the sides marked with letters in these triangles.

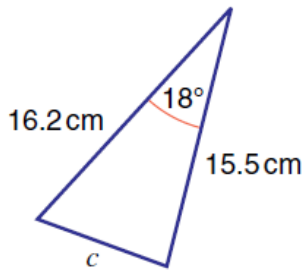
a



b

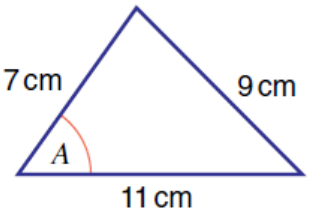


c

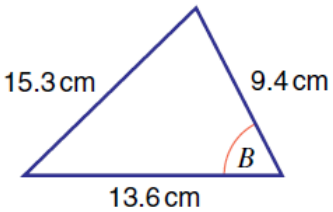


4. Calculate the size of each of the angles marked with a letter in these triangles.

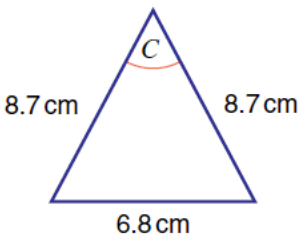
a



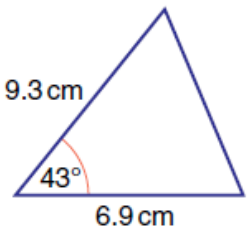
b



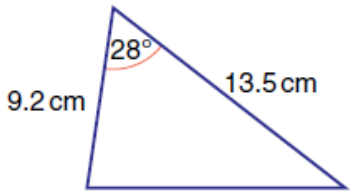
c



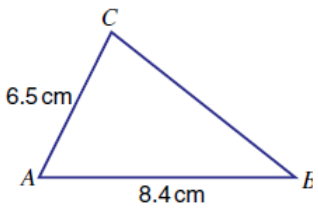
5. i



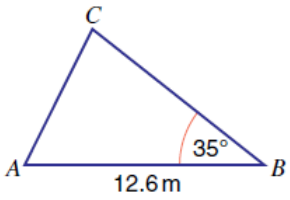
ii



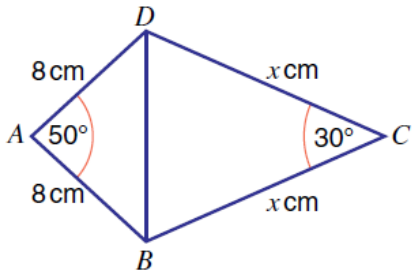
6. The area of triangle ABC is 15 cm^2
Angle A is acute.
Work out the size of angle A.



7. The area of triangle ABC is 60.7 m^2
Work out the length of BC.



8. $ABCD$ is a kite with diagonal DB.
   - **a** Calculate the length of DB.
   - **b** Calculate the size of angle BDC.
   - **c** Calculate the value of x.
   - **d** Calculate the length of AC.



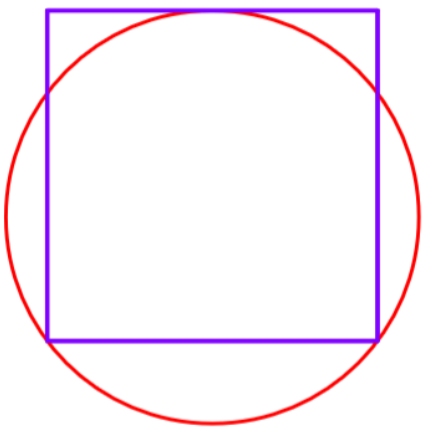
Mark your work.

Challenges

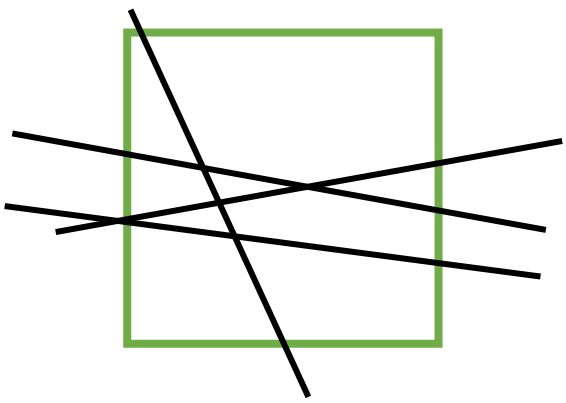
1. What is the value of the following sum?

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{14}+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{16}}=?$$

2. Which is longer – the perimeter of the square or the circumference of the circle?



3. I take an ordinary square and draw a random number of straight lines right across it;



I then colour in the square using as few possible colours as I can.
Note: regions that touch edge to edge **cannot** be the same colour.
Regions that only touch corner to corner **can** be the same colour,
What is the minimum number of colours I would ever have to use?
Can you prove it?

4. Which of the following are sometimes, always, or never true if a, b are positive integers?

   A. $\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}$

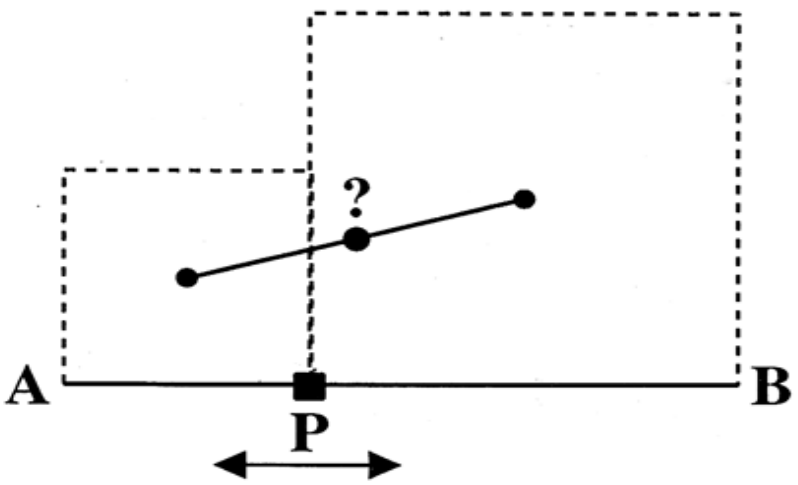
   B. $\frac{1}{a} + \frac{1}{b} = \frac{1}{ab}$

   C. $\frac{1}{a} - \frac{1}{b} = \frac{1}{ab}$

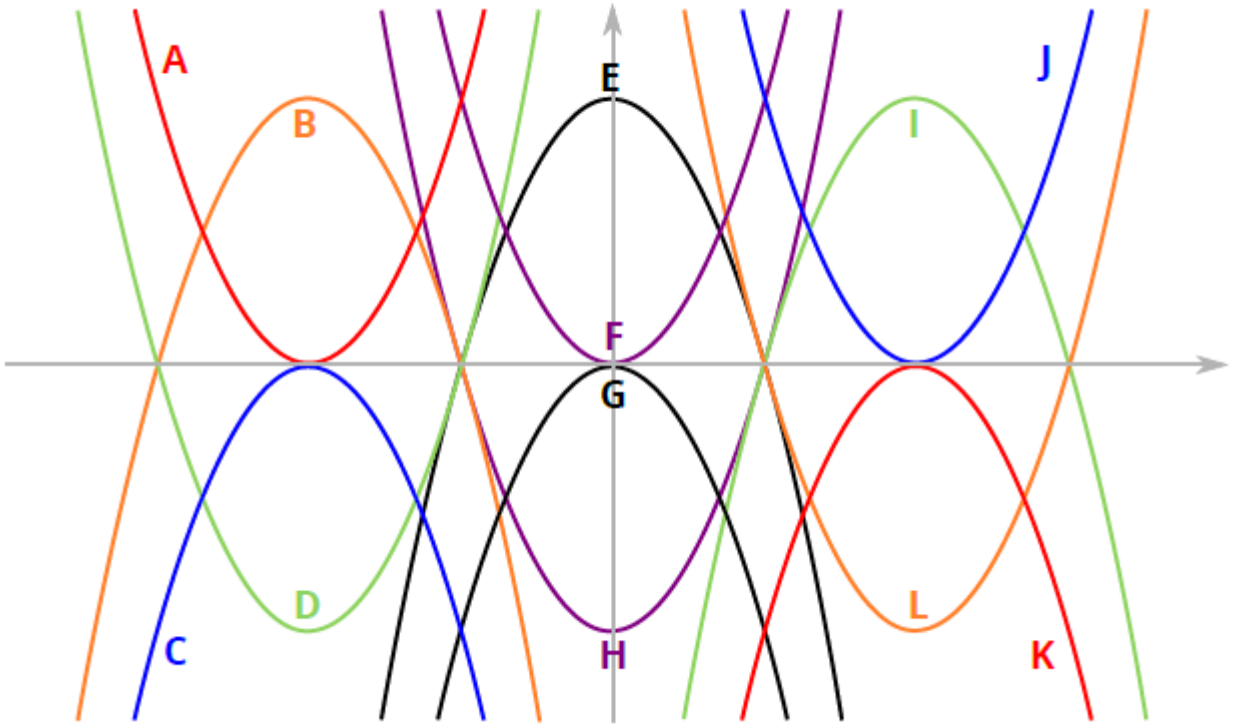
   D. $\frac{1}{a} \div \frac{1}{b} = \frac{1}{ab}$

   What if we allow decimals? What if we allow negatives?

5. Let **P** be a point moving along a line segment **AB**.
   Draw squares on segments **AP** and **PB** on the same side of **AB**.
   What is the set of Mid-Points of the line joining the centres of the squares as **P** moves along **AB**?



Problem - Which Parabola?



Given that two of the parabolas have the equations

$$y = x^2 - 12x + 27$$

and

$$y = -x^2 + 12x - 36,$$

can you find the equations of the other parabolas?

You might like to design your own pattern of parabolas, perhaps using Desmos. You could start from our design, if you like.

https://undergroundmathematics.org/quadratics/which-parabola

Problem – Nested Surds

For each card below, determine which non-negative values of a, b, c, and d, if any, make the equation true. These can be attempted in any order but you might find that some cards can help inform your decisions about others.

ⓐ $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$	ⓑ $\dfrac{\sqrt{a}}{\sqrt{b}}=\sqrt{\dfrac{a}{b}}$
ⓒ $\sqrt{23-6\sqrt{6-4\sqrt{2}}}=\sqrt{a}+\sqrt{b}$	ⓓ $a\sqrt{b}=\sqrt{ab}$
ⓔ $\dfrac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}=1$	ⓕ $\sqrt{a}-\sqrt{b}=\sqrt{a-b}$
ⓖ $\sqrt{a}+\sqrt{b}=\sqrt{a+b+\sqrt{4ab}}$	ⓗ $\dfrac{\sqrt{a}+b}{\sqrt{c}+d}=(\sqrt{a}+b)(\sqrt{c}-d)$
ⓘ $\sqrt{5+2\sqrt{6}}=\sqrt{a}+\sqrt{b}$	

https://undergroundmathematics.org/thinking-about-algebra/nested-surds

ANSWERS

Unit 1 - Answers

Practice Exercise 1.1

1. a) $(x+3)(x+4)$ b) $(x+4)(x+2)$ c) $(x+3)(x+2)$ d) $(x+7)(x+1)$
   e) $(x+2)^2$ f) $(x+3)(x+5)$ g) $(x+3)^2$ h) $(x+7)(x+4)$

2. a) $(x+4)(x-3)$ b) $(x+6)(x-1)$ c) $(x+5)(x-2)$ d) $(x+4)(x-1)$
   e) $(x+8)(x-6)$ f) $(x+8)(x-4)$ g) $(x+7)(x-5)$ h) $(x+11)(x-3)$

3. a) $(x-5)(x+2)$ b) $(x-5)(x+4)$ c) $(x-9)(x+3)$ d) $(x-3)(x+1)$
   e) $(x-4)(x+3)$ f) $(x-6)(x+2)$ g) $(x-7)(x+3)$ h) $(x-11)(x+5)$

4. a) $(x-3)^2$ b) $(x-5)(x-4)$ c) $(x-7)(x-2)$ d) $(x-11)(x-2)$
   e) $(x-8)(x-1)$ f) $(x-8)(x-4)$ g) $(x-7)(x+2)$ h) $(x-6)(x-8)$

5. a) $(x-8)(x-1)$ b) $(x+23)(x+1)$ c) $(x-7)(x+2)$ d) $(x-3)(x-4)$
   e) $(x+6)^2$ f) $(x-9)(x+7)$ g) $(x+13)(x+2)$ h) $(x+12)(x+5)$

Practice Exercise 1.2

1. a) $(x+5)(x-5)$ b) $(y+7)(y-7)$ c) $(w+10)(w-10)$ d) $(x+2)(x-2)$
   e) $(c+8)(c-8)$ f) $(x+1)(x-1)$ g) $(x+30)(x-30)$ h) $(y+3)(y-3)$
   i) $(16+x)(16-x)$ j) $(1+y)(1-y)$ k) $(9+x)(9-x)$ l) $(12+h)(12-h)$
   m) $(x+y)(x-y)$ n) $(a+c)(a-c)$ o) $(3x+5)(3x-5)$ p) $(2x+1)(2x-1)$

2. a) $(x^2+1)(x^2-1)$ b) $(x^2+1)(x^2-1)$ c) $(a^2+5)(a^2-5)$ d) $(x^2+y^2)(x^2-y^2)$
   e) $(h+p^2)(h-p^2)$ f) $(4x^2+7)(4x^2-7)$ g) $(y^3+6)(y^3-6)$ h) $(x^3+8)(x^3-8)$
   i) $(9p^2+x^3)(9p^2-x^3)$ j) $(12x^4+1)(12x^4-1)$

Practice Exercise 1.3

1. a) $(2x+5)(x+1)$ b) $(2x+5)(x+3)$ c) $(2x+5)(x+2)$
   d) $(3x+1)(x+4)$ e) $(3x+1)(x+1)$ f) $(3x+2)(x+2)$

2. a) $(3x+4)(x-1)$ b) $(7x-1)(x+3)$ c) $(2x-3)(x-5)$
   d) $(3x-2)(x-5)$ e) $(3x+2)(x-6)$ f) $(3x-4)(x+1)$

3. a) $(3x+2)(2x+3)$ b) $(3x+2)(3x+1)$ c) $(6x+1)(x+2)$
   d) $(8x+1)(x+5)$ e) $(3x+1)^2$ f) $(4x+3)(2x+5)$

4. a) $(3x+1)(3x-5)$ b) $(2x+1)(2x-3)$ c) $(4x-3)(x-2)$
   d) $(2x-1)(3x-2)$ e) $(10x-1)(x-9)$ f) $(4x-7)(x+8)$

Practice Exercise 1.4

1. a) $x =$ -4 or $x = -2$ b) $x = -4$ or $x = -3$ c) $x = -5$ or $x = -2$
   d) $x = -4$ or $x = 1$ e) $x = -2$ or $x = 4$ f) $x = 3$ or $x = 4$
   g) $x = 5$ h) $x = -5$ or $x = 9$ i) $x = -7$ or $x = 8$

2. a) $x = \pm 3$ b) $x = \pm 10$ c) $x = \pm 1$
   d) $x = \pm 12$ e) $x = \pm 8$ f) $x = \pm 0.5$

3. a) $x = -1$ b) $x = -7$ or $x = -1$ c) $x = 3$ or $x = 4$
   d) $x = -12$ or $x = -1$ e) $x = -2$ or $x = 5$ f) $x = 7$

Practice Exercise 1.5

1. a) $(x+4)^2 - 15$ b) $(x+5)^2 - 22$ c) $(x+1)^2 - 2$
   d) $(x-3)^2 - 19$ e) $(x-2)^2 - 17$ f) $(x-6)^2 - 33$
   g) $(x+7)^2 - 46$ h) $(x-1)^2 - 16$ i) $(x+2)^2 - 15$
   j) $(x+\frac{1}{2})^2 - \frac{35}{4}$ k) $(x+\frac{3}{2})^2 - \frac{5}{4}$ l) $(x-\frac{7}{2})^2 - \frac{57}{4}$

2. a) $2(x+4)^2 - 30$ b) $2(x+3)^2 - 21$ c) $3(x-2)^2 - 10$
   d) $4(x+\frac{3}{2})^2 - 14$ e) $2(x-\frac{3}{4})^2 - \frac{49}{8}$ f) $5(x-2)^2 + 10$

Practice Exercise 1.6

1. a) $x = -4$ or $x = -2$ b) $x = -6$ or $x = -4$ c) $x = -10$ or $x = -4$
   d) $x = -5$ or $x = 9$ e) $x = 5$ or $x = 7$ f) $x = 3$ or $x = -1$

2. a) $x = -4$ or $x = -1$ b) $x = -3$ or $x = 6$ c) $x = -4$ or $x = 3$
   d) $x = 3$ or $x = 4$ e) $x = 3$ or $x = 8$ f) $x = -3$ or $x = 10$

3. a) $(x+2)^2 - 7 = 0$
   $(x+2)^2 = 7$
   $x + 2 = \pm\sqrt{7}$
   $x = -2 \pm \sqrt{7}$

   b) $(x+3)^2 - 19 = 0$
   $(x+3)^2 = 19$
   $x + 3 = \pm\sqrt{19}$
   $x = -3 \pm \sqrt{19}$

   c) $(x-1)^2 - 6 = 0$
   $(x-1)^2 = 6$
   $x - 1 = \pm\sqrt{6}$
   $x = 1 \pm \sqrt{6}$

   d) $(x-5)^2 - 24 = 0$
   $(x-5)^2 = 24$
   $x - 5 = \pm\sqrt{24}$
   $x = 5 \pm \sqrt{24}$

   e) $(x+4)^2 - 13 = 0$
   $(x+4)^2 = 13$
   $x + 4 = \pm\sqrt{13}$
   $x = -4 \pm \sqrt{13}$

   f) $(x-4)^2 - 38 = 0$
   $(x-4)^2 = 38$
   $x - 4 = \pm\sqrt{38}$
   $x = 4 \pm \sqrt{38}$

4. a) $5(x^2+6x)-10=0$
$5(x+3)^2-55=0$
$5(x+3)^2=55$
$(x+3)^2=1$
$x+3=\pm\sqrt{1}$
$x=-4$ or $x=-2$

b) $2\left(x^2+\frac{7}{2}x\right)+3=0$
$2(x+\frac{7}{4})^2-\frac{25}{8}=0$
$2(x+\frac{7}{4})^2=\frac{25}{8}$
$(x+\frac{7}{4})^2=\frac{25}{16}$
$x+\frac{7}{4}=\pm\sqrt{\frac{25}{16}}$
$x=-\frac{1}{4}$ or $x=-\frac{11}{4}$

c) $3(x^2+4x)-2=0$
$3(x+2)^2-14=0$
$3(x+2)^2=14$
$(x+2)^2=\frac{14}{3}$
$x+2=\pm\sqrt{\frac{14}{3}}$
$x=-2\pm\sqrt{\frac{14}{3}}$

d) $2\left(x^2-\frac{3}{2}x\right)-7=0$
$2\left(x-\frac{3}{4}\right)^2-\frac{65}{8}=0$
$2\left(x-\frac{3}{4}\right)^2=\frac{65}{8}$
$\left(x-\frac{3}{4}\right)^2=\frac{65}{16}$
$x-\frac{3}{4}=\pm\sqrt{\frac{65}{16}}$
$x=\frac{3\pm\sqrt{65}}{4}$

e) $5\left(x^2+\frac{2}{5}x\right)-8=0$
$5\left(x+\frac{1}{5}\right)^2-\frac{41}{5}=0$
$5\left(x+\frac{1}{5}\right)^2=\frac{41}{5}$
$\left(x+\frac{1}{5}\right)^2=\frac{41}{25}$
$x+\frac{1}{5}=\pm\sqrt{\frac{41}{25}}$
$x=\frac{-1\pm\sqrt{41}}{25}$

f) $10\left(x^2-\frac{1}{5}x\right)-1=0$
$10\left(x-\frac{1}{10}\right)^2-\frac{11}{10}=0$
$10\left(x-\frac{1}{10}\right)^2=\frac{11}{10}$
$\left(x-\frac{1}{10}\right)^2=\frac{11}{100}$
$x-\frac{1}{10}=\pm\sqrt{\frac{11}{100}}$
$x=\frac{1\pm\sqrt{11}}{100}$

Practice Exercise 1.7

1. (a) -0.2 and -4.8 (b) -0.3 and -3.2 (c) -0.5 and -1.5
(d) 1.2 and -3.2 (e) 0.8 and -2.1 (f) 1.3 and -3.8
(g) 0.6 and 3.4 (h) 0.2 and 0.6 (i) 0.5 and 2.9
(j) -2.9 and 3.9 (k) 8.4 and -2.4 (l) 2.4 and -1.9

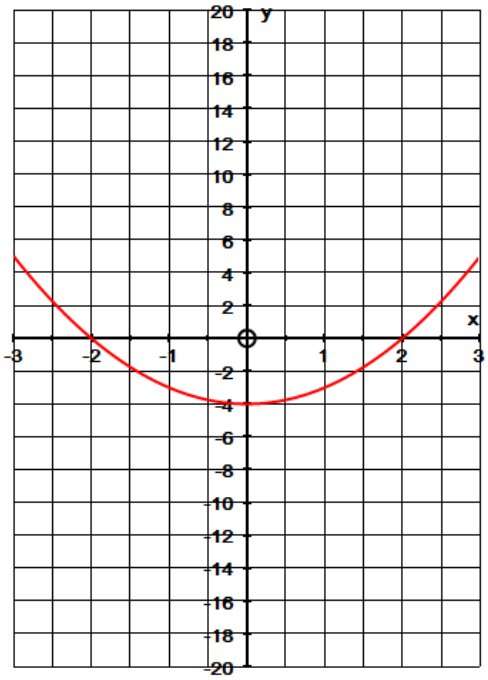
2. (a) 2.2 and -9.2 (b) 7.3 and -2.8 (c) 1.5 and -2.2
(d) 3.4 and -2.4 (e) 1.5 and -8.5 (f) 3.3 and -1.3

Unit 2 Answers

Practice Exercise 2.1

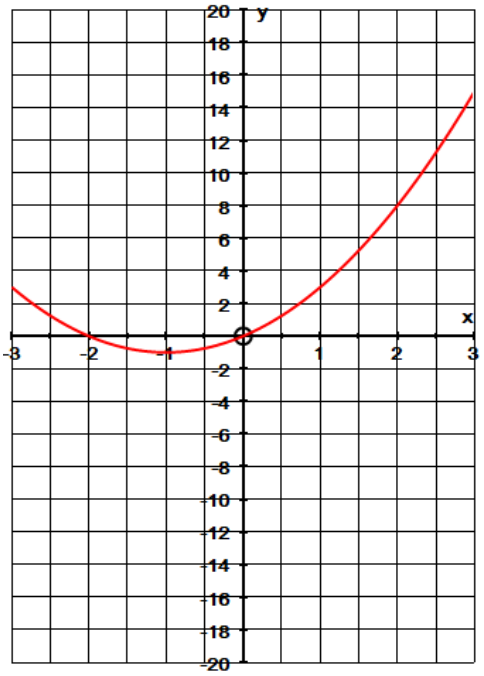
1. $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5



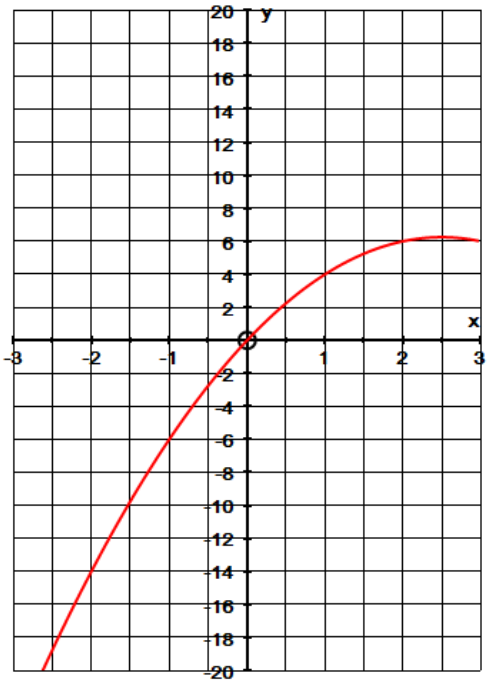
2. $y = x^2 + 2x$

x	-3	-2	-1	0	1	2	3
y	3	0	-1	0	3	8	15



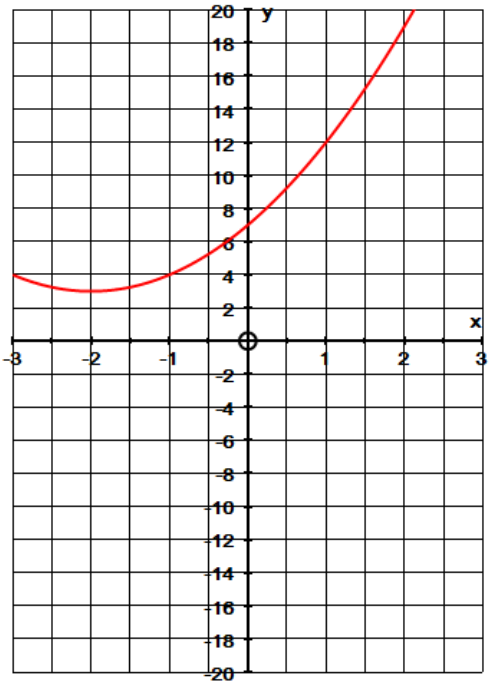
3. $y = 5x - x^2$

x	-3	-2	-1	0	1	2	3
y	-24	-14	-6	0	4	6	6



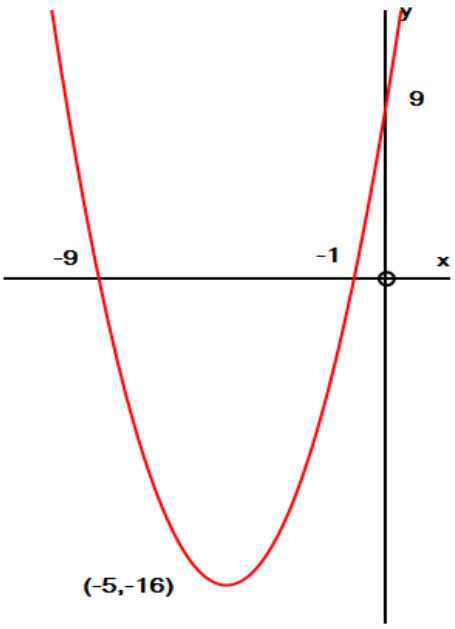
4. $y = x^2 + 4x + 7$

x	-3	-2	-1	0	1	2	3
y	4	3	4	7	12	19	28

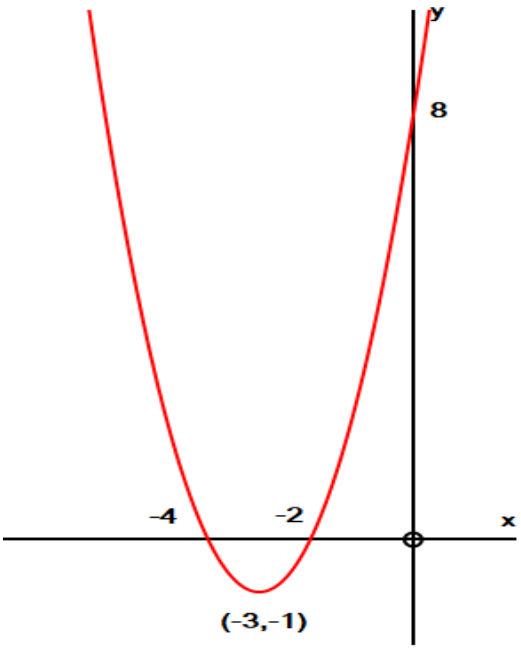


Practice Exercise 2.2

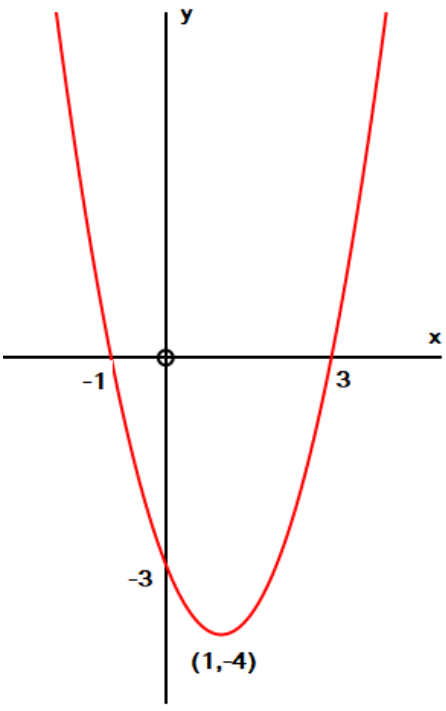
1.



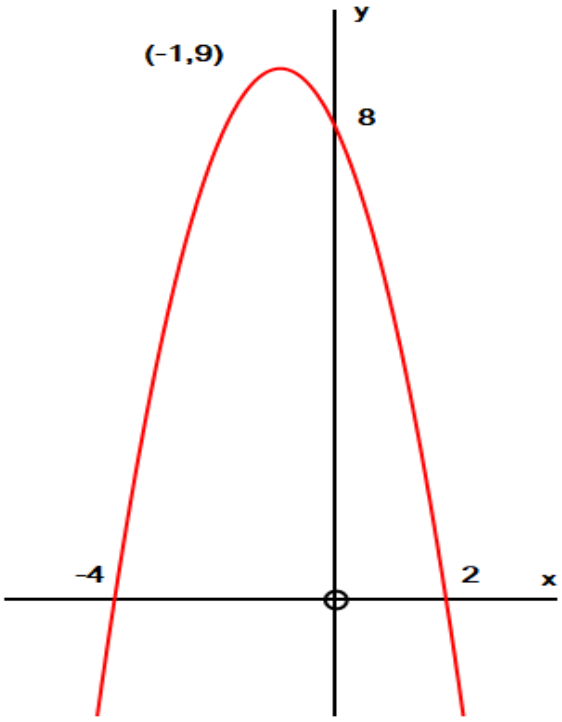
2.



3.



4.



Practice Exercise 2.3

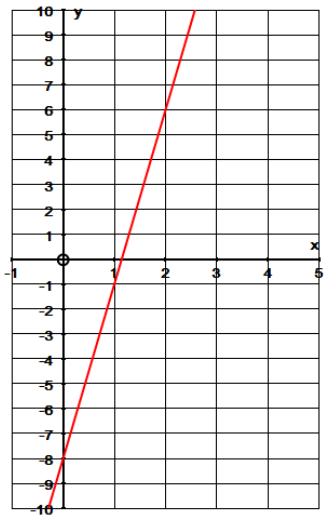
1. $y = (x - 4)^2$-5
2. $y = (x - 3)(x + 5)$
3. $y = (x + 2)^2 + 3$
4. $y = (x - 1)(x - 3)$

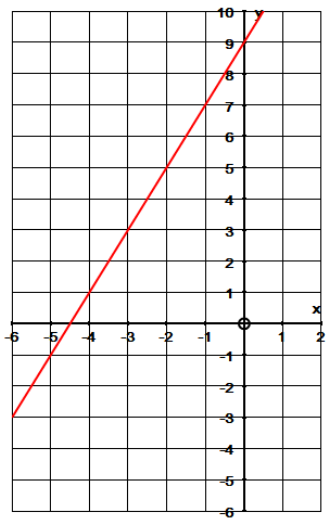
Unit 3 - Answers

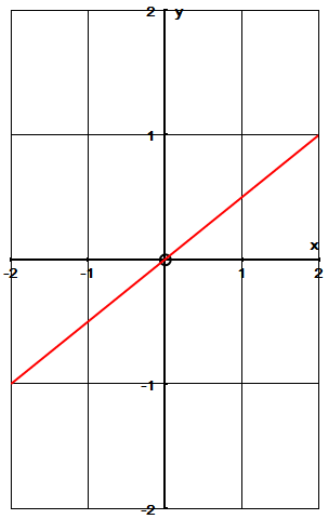
Practice Exercise 3.1

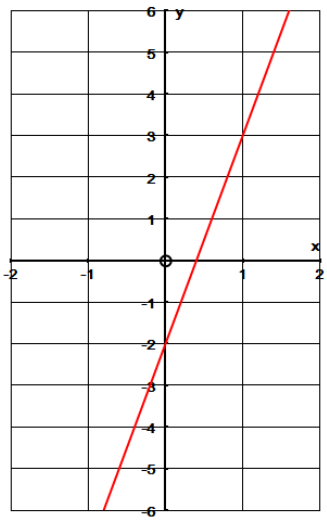
1. a) Gradient = 7, y-intercept = -8
   b) Gradient = 2, y-intercept = 9
   c) Gradient = 0.5, y-intercept = 0

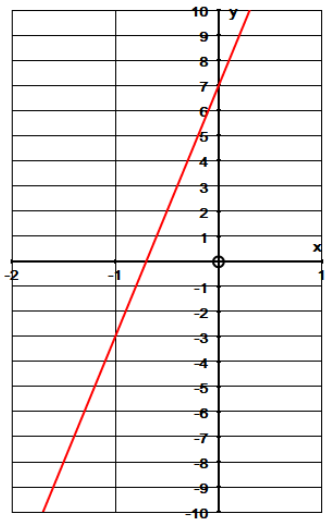
2. a) $y = 5x - 2$
   b) $y = 10x + 7$
   c) $y = -3x + 1$

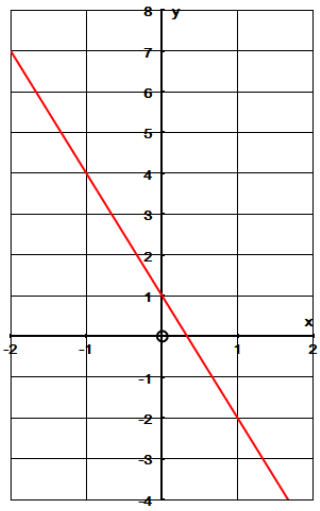
3. 1a) 

1b) 

1c) 

2a) 

2b) 

2c) 

4. a) $y = 3x - 2$
   b) $y = -x + 3$
   c) $y = -2x + 4$
   d) $y = 0.5x + 2$

Practice Exercise 3.2

1.	$y = -x + 13$	Gradient = -1, y-intercept = 13
2.	$y = 4x + 6$	Gradient = 4, y-intercept = 6
3.	$y = 2x + 7$	Gradient = 2, y-intercept = 7
4.	$y = -2x + \frac{5}{2}$	Gradient = -2, y-intercept = $\frac{5}{2}$
5.	$y = -\frac{1}{4}x + 5$	Gradient = $-\frac{1}{4}$, y-intercept = 5
6.	$y = 3x - \frac{3}{2}$	Gradient = 3, y-intercept = $-\frac{3}{2}$
7.	$y = 18x - 9$	Gradient = 18, y-intercept = -9
8.	$y = \frac{6}{7}x + 1$	Gradient = $\frac{6}{7}$, y-intercept = 1
9.	$y = \frac{6}{5}x - \frac{12}{5}$	Gradient = $\frac{6}{5}$, y-intercept = $-\frac{12}{5}$
10.	$y = \frac{6}{5}x - \frac{4}{5}$	Gradient = $\frac{6}{5}$, y-intercept = $-\frac{4}{5}$

Practice Exercise 3.3

1. $y = 3x - 1$
2. $y = 7x + 21$
3. $y = -2x + 8$
4. $y = -10x - 25$
5. $y = 3x + 4$
6. $y = 2x + 6$
7. $y = 2x - 2$
8. $y = \frac{1}{2}x - \frac{17}{2}$
9. $y = 4x - 11$
10. $y = -x - 4$
11. $y = x + 7$
12. $y = \frac{1}{2}x + 7$

Practice Exercise 3.4

1. $y = -\frac{1}{2}x + 4$
2. $y = \frac{1}{3}x + \frac{5}{3}$
3. $y = -2x + 18$
4. $y = -8x - 4$
5. $y = \frac{3}{2}x - \frac{19}{2}$

Unit 4 – Answers

Practice Exercise 4.1a

1 **a** 2^9 **b** 2^7 **c** 2^8 **d** 2^7 **e** 2^{10}

2 **a** 3^2 **b** 3^3 **c** 3^3 **d** 3^4 **e** 3^6

3 **a** 4^2 **b** 5^5 **c** 3^6 **d** 6^7 **e** 10^2

4 **a** 5 **b** 3 **c** 5 **d** 4 **e** 9

5 **a** 9 **b** 16 **c** 8 **d** 10^6 **e** 6

6 **a** 3^4 **b** 3^6 **c** 3^4 **d** 3^2 **e** 3^5

7 **a** 2^2 **b** 3^3 **c** 5^2 **d** 10^4 **e** 4^4

8 **a** 5 **b** 1 **c** 16 **d** 2 **e** 9

9 **a** 3 **b** 5 **c** 2 **d** 4 **e** 3

Practice Exercise 4.1b

1 **a** x^5 **b** y^8 **c** n^7 **d** q^8 **e** x^2
f y^4 **g** p **h** q^6 **i** y^8 **j** q^2

2 **a** $3x^7$ **b** $8p^5$ **c** $20p^2$ **d** $16r^9$ **e** $3y^3$
f $2q$ **g** $4x$ **h** 2 **i** $6y^8$ **j** 15

3 **a** **i** x^3 **ii** x^3 **iii** $2x^3$ **b** $2y^4$

4 **a** $3a^7b^2$ **b** $8a^4b^5$ **c** $10p^6q^6$
d $3x^5y^4$ **e** $4b^4$ **f** $10pq^{-1}$

5 **a** 4 **b** 1

6 **a** x^{-4} **b** x^{-1} **c** x^{-7} **d** x^{-3} **e** x^2

7 **a** x^{15} **b** $16y^8$ **c** $a^{10}b^{20}$ **d** $27a^9b^3$
e x^{-2} **f** $16y^{-4}$ **g** a^{12} **h** $\frac{1}{4}b^6$

8 **a** 1 **b** 1 **c** $a^{-18}b^9$

Practice Exercise 4.2a

1 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{1}{5}$ **d** $\frac{1}{1000}$
e 1 **f** $\frac{2}{5}$ **g** 3 **h** $\frac{9}{4}$

2 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** 25 **d** $\frac{1}{36}$ **e** $\frac{1}{8}$

3 **a** $\frac{1}{4}$ **b** 27 **c** 4 **d** $\frac{1}{10}$ **e** $\frac{1}{25}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{5}$ **d** 16 **e** 32

5 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** 5 **d** 64 **e** 1

6 **a** -3 **b** -3 **c** -2 **d** -6

7 **a** 700 **b** 200 000 **c** 4900 **d** 0.8 **e** 0.0069

8 **a** 3 **b** 4 **c** -1 **d** -2

9 **a** 0.8 **b** 6.25 **c** 6.25 **d** 0.064 **e** 0.0064

10 **a** 2.22 **b** 0.198 **c** 1.13 **d** 30 800

Practice Exercise 4.2b

1 **a** 3 **b** 5 **c** 10 **d** 2 **e** $\frac{1}{2}$

2 **a** 3 **b** 10 **c** -4 **d** 5 **e** $\frac{1}{5}$

3 **a** $\frac{1}{16}$ **b** $\frac{1}{9}$ **c** $\frac{4}{9}$ **d** $\frac{4}{25}$ **e** $\frac{27}{64}$

4 **a** 9 **b** 100 **c** 16 **d** 8 **e** 125

5 **a** $\frac{1}{5}$ **b** $\frac{1}{3}$ **c** $\frac{1}{3}$ **d** $\frac{1}{4}$ **e** $\frac{1}{512}$

6 **a** $-\frac{1}{2}$ **b** $\frac{5}{2}$ **c** $\frac{11}{3}$

Practice Exercise 4.3

1 **a** 2 **b** 3 **c** 5 **d** 4 **e** 6

2 **a** $3+2\sqrt{3}$ **b** $5+3\sqrt{3}$ **c** $3+\sqrt{5}$
d -12 **e** $7-4\sqrt{3}$

3 **a** $\frac{\sqrt{2}}{2}$ **b** $\frac{\sqrt{5}}{5}$ **c** $\frac{2\sqrt{7}}{7}$ **d** $\frac{3\sqrt{2}}{2}$ **e** $\frac{5\sqrt{11}}{11}$

4 **a** $\frac{\sqrt{6}}{3}$ **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{\sqrt{10}}{2}$ **d** $\sqrt{2}$ **e** $2\sqrt{5}$

5 **a** $1+\sqrt{2}$ **b** $\sqrt{2}-1$ **c** $1+2\sqrt{5}$
d $\sqrt{5}-1$ **e** $2\sqrt{7}+1$

6 4 cm

7 $3+2\sqrt{2}\,\text{cm}^2$

8 **a** **i** 14 cm **ii** $7+\sqrt{5}\,\text{cm}^2$

Unit 5 – Answers

Practice Exercise 5.1

1. (a) x=2, y=6 (b) x=1, y=3 (c) x=3, y=2

   (d) x=15, y=6 (e) x=5, y=6 (f) x=3, y=0.5

2. (a) x=5, y=4 (b) x=5, y=2 (c) x=4, y=9

   (d) x=11, y=10 (e) x=7.5, y=3.5 (f) x=7, y=1

3. (a) x=5, y=2 (b) x= -4, y= 3 (c) x=2, y=4

   (d) x=8, y=2 (e) x=-1, y=5 (f) x=-4, y=2

Practice Exercise 5.2

1. (a) x = -5 and y = -2, x = 1 and y = 4

   (b) x = -3 and y = -8, x = 3 and y = -2

   (c) x = 1 and y = 1, x = 3 and y = 5

   (d) x = -4 and y = -3, x = 1 and y = 12

   (e) x = -4 and y = 29, x = -2 and y = 7

   (f) x = -0.5 and y = -0.75, x = 2 and y = 8

2. (a) x = -5 and y = 9, x = 1 and y = 3

   (b) x = 2 and y = 5, x = 5 and y = 2

   (c) x = 2 and y = 3, x = 3 and y = 2

   (d) x = -2 and y = 0, x = 3 and y = 10

   (e) x = -5 and y = -2, x = -2 and y = -5

   (f) x = -6 and y = 1, x = 2 and y = -3

3. (a) x = 3 and y = 1, x = 6.333... and y = -5.666..

(b) x = -4.284 and y = -1.284, x = 1.284 and y = 4.284

(c) x = -0.2915 and y = -0.2915, x = 10.2915 and y = 10.2915

(d) x = 1.14 and y = -2.72, x = 2.19 and y = -0.613

(e) x = -4.46 and y = 8.42, x = 1.46 and y = -3.42

(f) x = -4.46 and y = -6.46, x = 2.46 and y = 0.46

Practice Exercise 5.3

1 a $x = 0, y = 0$ or $x = 1, y = 2$
b $x = 1, y = 4$ or $x = -3, y = 0$
c $x = 4, y = 11$ or $x = -1, y = -4$
d $x = 1, y = 5$ or $x = -5, y = -9$

2 a $x = 3, y = 13$ or $x = -1.5, y = -0.5$
b $x = 1, y = 1$ or $x = -\frac{4}{3}, y = \frac{10}{3}$
c $x = 1, y = 3$ or $x = -\frac{5}{2}, y = 10$
d $x = 2, y = -1$ or $x = 1, y = -2$
e $x = 4, y = 16$ or $x = -2, y = 4$
f $x = 2, y = 3$ or $x = -\frac{8}{3}, y = \frac{55}{9}$

Practice Exercise 5.4

1. **i** $x = 1, y = 2$ **ii** $x = 2, y = 2$ **iii** $x = 1, y = -1$

2 $x = -1, y = 1$ **3** $x = -2, y = -4$ **4** $x = -2, y = 3$

5 $x = -3, y = -2$ **6** $x = 3, y = 1$

7. a) $x = 1, y = 3$ or $x = -3, y = 3$ b) $x = 5, y = 5$ or $x = -1, y = 5$
c) $x = 1, y = -4$ or $x = 4, y = -4$ d) $x = -2, y = -1$ or $x = -0.5, y = -1$

8. a) $x = 3, y = 9$ or $x = -2, y = 4$ b) $x = 2, y = 2$ or $x = -1, y = -1$
c) $x = 1, y = 2$ or $x = -0.5, y = 0.5$ d) $x = -1, y = 5$ or $x = 0.5, y = 3.5$
e) $x = -1, y = -1$ or $x = -5, y = -5$ f) $x = -2, y = -1$ or $x = -0.25, y = 0.125$

Unit 6 – Answers

Practice Exercise 6.1

1. (a) $y = c - w$ (b) $y = m + p$ (c) $y = s - m$

(d) $y = n + 2g$ (e) $y = \frac{c}{3}$ (f) $y = \frac{w}{a}$

(g) $y = cw$ (h) $y = 2ac$ (i) $y = a - p$

(j) $y = c + k$ (k) $y = \sqrt{s}$ (l) $y = \sqrt[y]{x}$

2. (a) $x = 2A - y$

(b) $x = \frac{A - \pi r^2}{2\pi r}$

(c) $x = \sqrt{\frac{T + y}{3}}$

(d) $x = \frac{m}{as}$

(e) $x = \frac{2S - 2uy}{y^2}$

(f) $x = \frac{4}{3}w - 4t$

(g) $x = dj - 3$

(h) $x = \frac{t}{g} + 2$

(i) $x = \frac{\sqrt{\frac{p}{3}} - y}{2}$

(j) $x = 8w - \frac{1}{2}a$

3. (a) $m = -17y$

(b) $m = 7a - 6$

(c) $m = -\frac{2}{5}c$

(d) $m = \frac{2}{3}(a - 2c)$

(e) $m = \frac{c(2 - a)}{a - 6}$

(f) $m = \frac{n(w + x)}{x - w}$

(g) $m = \frac{3c + 8f}{7}$

(h) $m = \frac{4 - 5y}{y - 1}$

(i) $m = \frac{y}{3t - a^2}$

(j) $m = \frac{cr + 7r - 5}{3}$

Unit 7 – Answers

Practice Exercise 7.1

1. (a) $x \leq 4$ (b) $x > 7$ (c) $x < 6$ (d) $x \geq 14$

   (e) $x \leq 8$ (f) $x > 90$ (g) $x \geq 7$ (h) $x > 13$

2. (a) $x \geq 11$ (b) $x < 5$ (c) $x \leq 3.4$ (d) $x < 6.5$

   (e) $x \geq 10$ (f) $x \leq -2$

3. (a) $x > 4$ (b) $9.5 \geq x$ (c) $x < 2.5$ (d) $x \geq 4$

4. (a) $3 < x < 7$ (b) $2 \leq x \leq 3.5$ (c) $\frac{1}{3} \leq x \leq 3$ (d) $20 < x < 30$

   (e) $3 \leq x \leq 11$ (f) $-8 \leq x < 4$

Practice Exercise 7.2

1. (a) $1 < x < 4$
   (b) $-1 < x < 2$
   (c) $-7 \leq x \leq -3$
   (d) $-4 \leq x \leq 5$
   (e) $x < 0$ or $x > 9$
   (f) $x < -6$ or $x > 5$
   (g) $x \leq -10$ or $x \geq -1$
   (h) $x \leq -7$ or $x \geq 7$
   (i) $-8 < x < -2$

2. (a) $x < -3$ or $x > -2$
   (b) $-5 < x < -2$
   (c) $-1 \leq x \leq 5$
   (d) $x < -6$ or $x > 4$
   (e) $x \leq 2$ or $x \geq 4$
   (f) $-4 < x < 1$
   (g) $x < -7$ or $x > 8$
   (h) $-6 < x < -3$
   (i) $2 \leq x \leq 11$

3. (a) $-3 < x < 5$

(b) $x < -4$ or $x > -1$

(c) $-6 < x < 6$

(d) $x < -11$ or $x > 11$

(e) $-4 \leq x \leq 3$

(f) $4 < x < 10$

4. (a) $x < -1$ or $x > -0.4$

(b) $-3 < x < 0.3333\ldots$

(c) $x < 0.5$ or $x > 4$

(d) $x \leq -0.25$ or $x \geq 1$

(e) $1 < x < 1.1666666..$

(f) $-2 \leq x \leq 1.5$

(g) $x < 0.75$ or $x > 2$

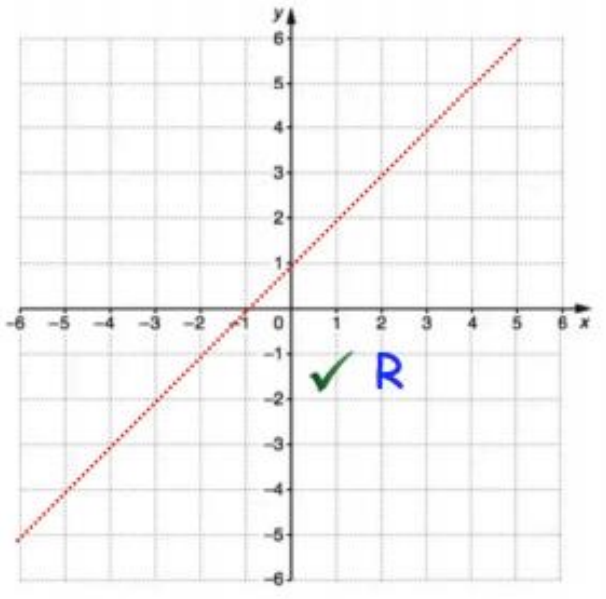
(h) $x \leq 0.75$ or $x \geq 6$

(i) $-1.666\ldots < x < 1.4$

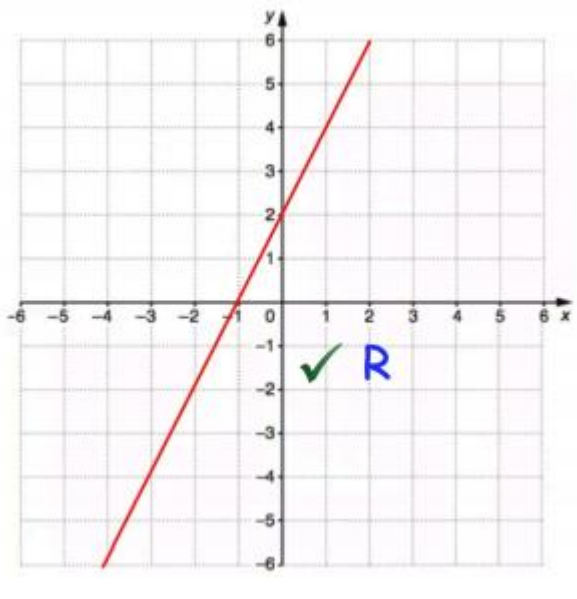
Practice Exercise 7.3

1.

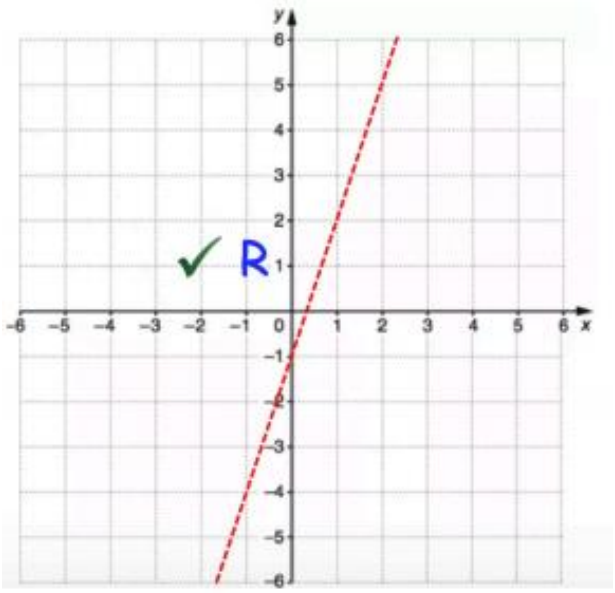
a)



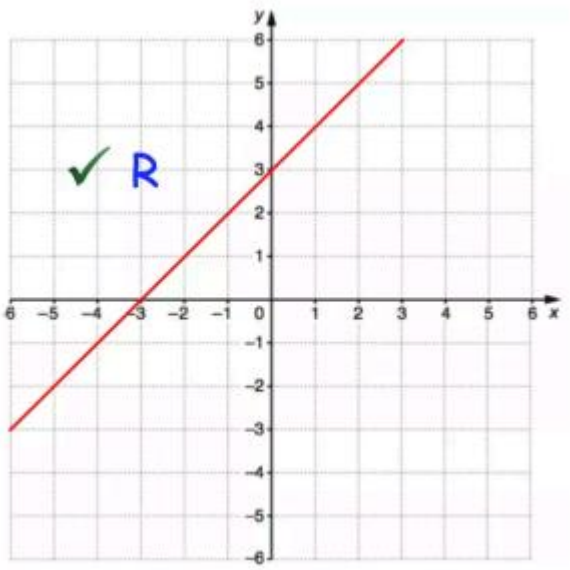
b)



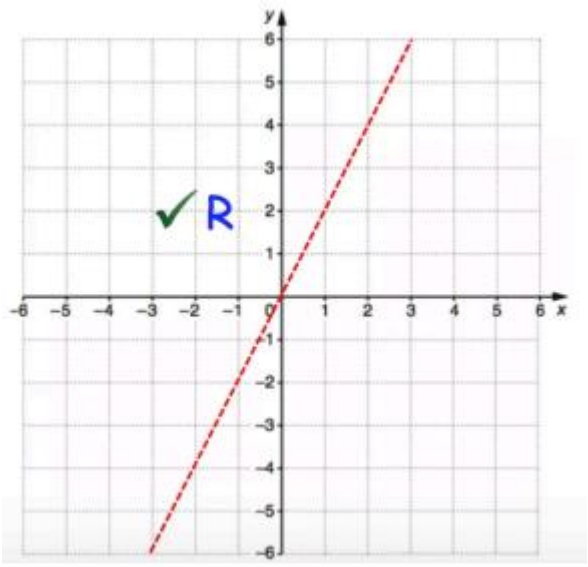
c)



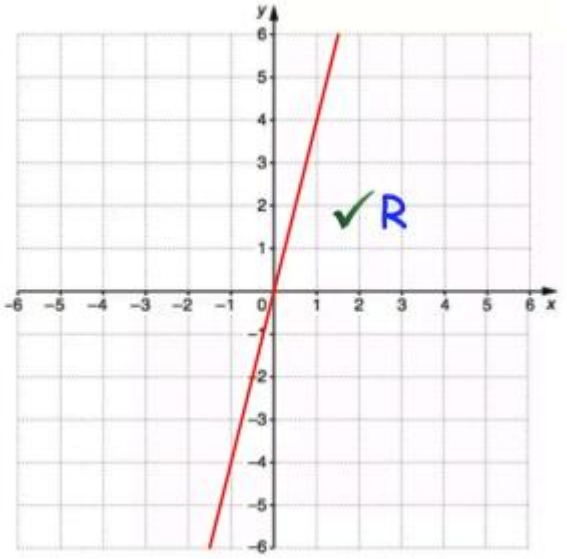
d)



e)

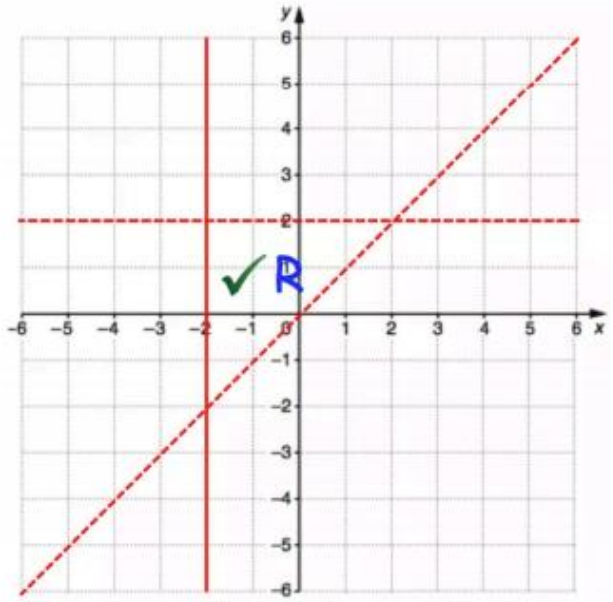


f)

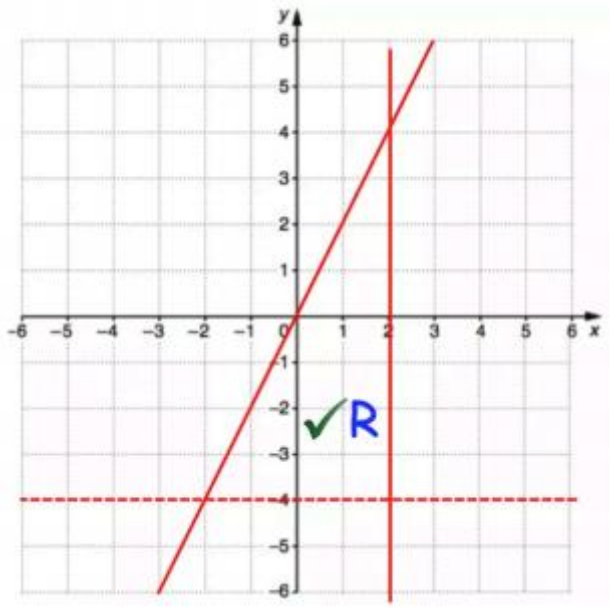


2.

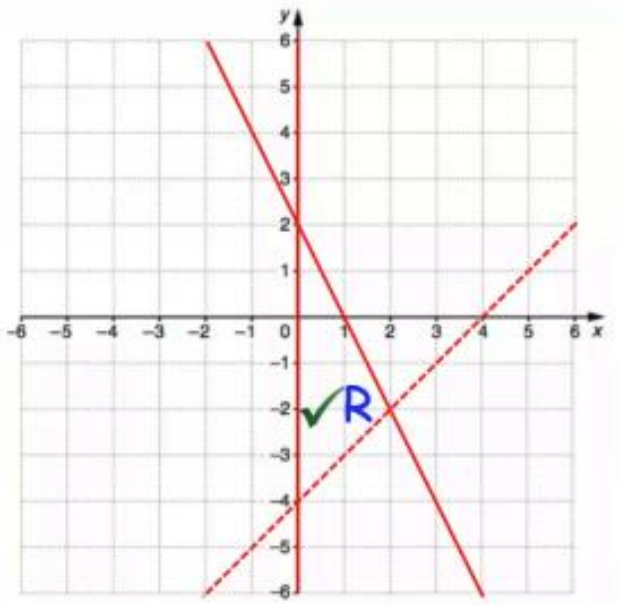
a)



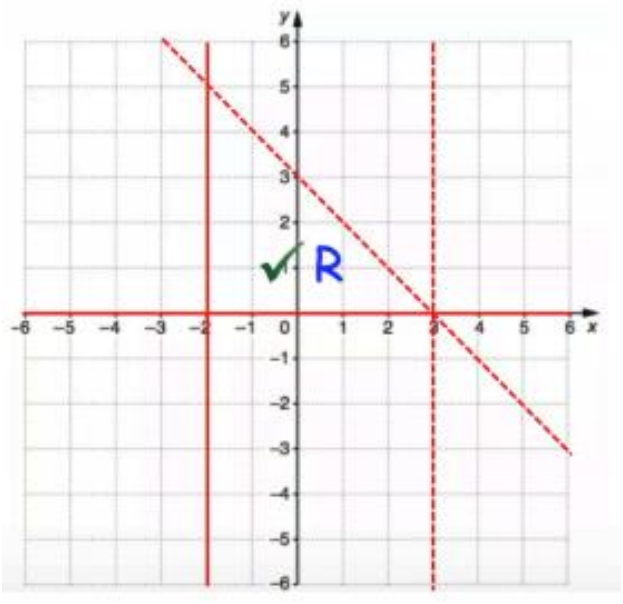
b)



c)



d)



3. (a) $y < x + 1$

(b) $y \geq x - 1$

(c) $y > 2x$

(d) $y \leq 2x + 4$

(e) $y < -3x + 6$

(f) $y > 4x + 1$

4. (a) $x > 2, \quad y \leq 4, \quad y \geq x - 1$

(b) $y \geq -1, \quad y \leq -2x + 6, \quad y < 4x$

Unit 8 – Answers

Practice Exercise 8.1

1. **a** 5.47 cm **b** 17.1 cm **c** 11.6 cm
   d 7.79 cm **e** 25.5 cm **f** 9.18 cm

2. **a** 24.2 cm **b** 29.6 cm

3. **a** 37.7° **b** 46.2° **c** 19.7°
   d 45.6° **e** 47.1° **f** 43.6°

4. **a** 13 cm
   b **i** 19.8 cm **ii** 19.2 cm
   c **i** 67.4° **ii** 38.7°

5. **a** 15.6 m
   b **i** 26.6° **ii** 36.9° **iii** 22.6°

Practice Exercise 8.2

1. **a** 8.06 cm **b** 7.19 cm **c** 6.35 cm

2. **a** 45.0° **b** 63.6° **c** 23.6°

3. **a** 8.79 cm **b** 12.6 cm **c** 5.01 cm

4. **a** 54.7° **b** 81.2° **c** 46.0°

5. **i** 21.9 cm^2 **ii** 29.2 cm^2

6. 33.3°

7. 16.8 m^2

8. **a** 6.76 cm **b** 75° **c** 13.1 cm **d** 19.9 cm

Challenge Hints and Answers

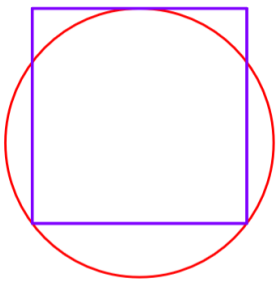
1. What is the value of the following sum?

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{14}+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{16}}=?$$

HINT: Try rationalising the surds…

ANSWER: 3

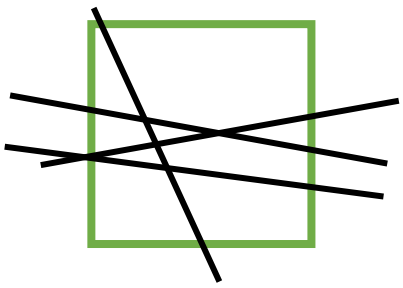
2. Which is longer – the perimeter of the square or the circumference of the circle?



HINT: Try letting the length of the side of the square be 8. Draw in the radius…

ANSWER: The Square is JUST bigger.

3. I take an ordinary square and draw a random number of straight lines right across it;



I then colour in the square using as few possible colours as I can. Note: regions that touch edge to edge **cannot** be the same colour. Regions that only touch corner to corner **can** be the same colour,

What is the minimum number of colours I would ever have to use?

Can you prove it?

Hint: Try doing the colouring! Start with a square and put four lines across at random and colour.

Hint for Proof: Try 1 line, 2, 3 etc and see what happens.

ANSWER: only 2 colours are ever needed.

4. Which of the following are sometimes, always, or never true if a, b are positive integers?

   A. $\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}$

   B. $\frac{1}{a} + \frac{1}{b} = \frac{1}{ab}$

   C. $\frac{1}{a} - \frac{1}{b} = \frac{1}{ab}$

   D. $\frac{1}{a} \div \frac{1}{b} = \frac{1}{ab}$

   What if we allow decimals? What if we allow negatives?

ANSWER:

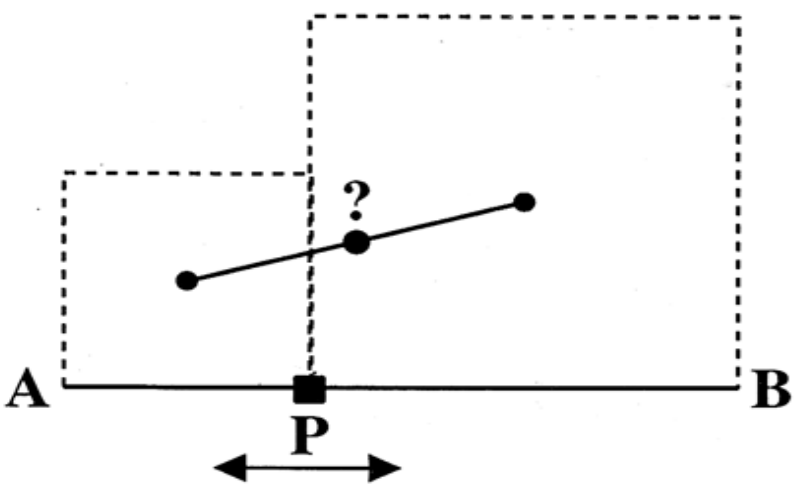
A is always true.

B is never true for integer values (only true if a+b=1), sometimes true for Negatives or Decimals.

C is sometimes true for integer values, if b-a=1

D is sometimes true (if b=1!) (Or, indeed if b=-1)

5. Let **P** be a point moving along a line segment **AB**.
   Draw squares on segments **AP** and **PB** on the same side of **AB**.
   What is the set of Mid-Points of the line joining the centres of the squares as **P** moves along **AB**?



HINT: Try to do this *without drawing first!* What shape do you think it will be?

ANSWER: A straight line of height $\frac{AB}{4}$

Problem - Which Parabola?

Hints and Answers

HINTS:

- Can you factorise these equations? What does this tell you about the graphs of these two functions?

- What is the difference between the graphs of $y = x^2 + bx + c$ and $y=-x^2 - bx - c$? Can you spot any graphs in the picture that might satisfy such a relation?

- If you know that a graph has equation $y = ax^2 + bx + c$, what is the equation of its reflection in the y-axis?

ANSWERS:

Parabola	Equation
A	$y = x^2 + 12x + 36$
B	$y = -x^2 - 12x - 27$
C	$y = -x^2 - 12x - 36$
D	$y = x^2 + 12x + 27$
E	$y = -x^2 + 9$
F	$y = x^2$
G	$y = -x^2$
H	$y = x^2 - 9$
I	$y = -x^2 + 12x - 27$
J	$y = x^2 - 12x + 36$
K	$y = -x^2 + 12x - 36$
L	$y = x^2 - 12x + 27$

For more explanation see: https://undergroundmathematics.org/quadratics/which-parabola/solution

Problem – Nested Surds

Hints and Answers

HINTS:

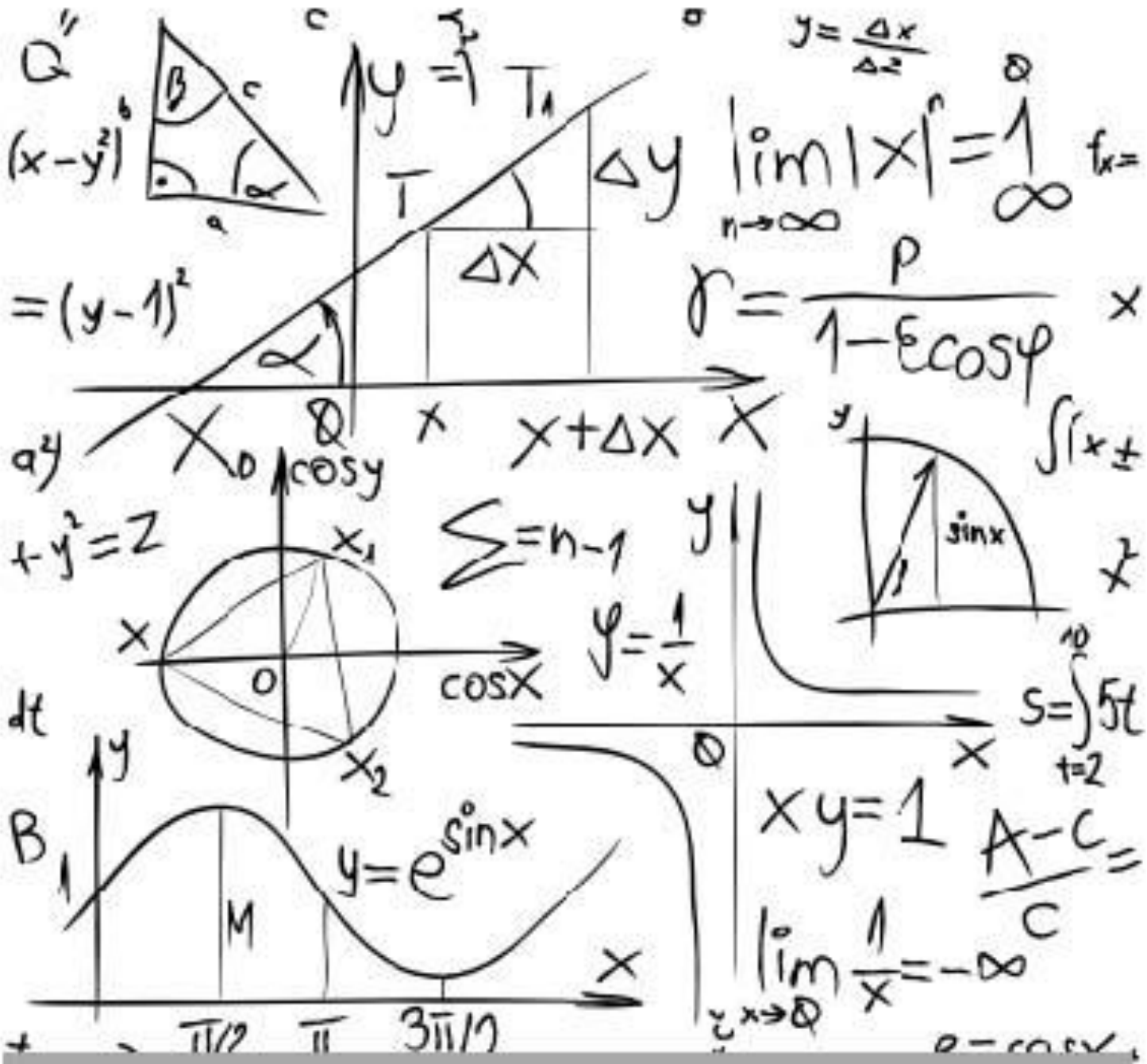
Have you tried substituting some numbers into the equations? This will give you a feeling for how the equation is behaving and may help you to decide what to do next.

Can you find equations that look familiar? Perhaps start here.

Can you sort the cards into pairs or groups of equations with similar properties?

ANSWERS:

See https://undergroundmathematics.org/thinking-about-algebra/nested-surds/solution



Ripley St Thomas

A-Level Maths

Transition Piece 1

Name: _____

This section of the Bridging Unit is **compulsory** and **must** be handed in during your first Mathematics lesson.

You are expected to submit this homework in during the first Mathematics lesson after the summer holidays. In the new academic year you will sit a test as part of the transition period. The test will focus on many of the skills included in this booklet. The knowledge and skills assessed in this booklet, and on the induction test, are required for success in A-Level Mathematics. Therefore, it is essential that you are confident in all the skills included in this booklet.

Success in A-Level Mathematics relies on an excellent attitude to learning and commitment to your studies. If you are struggling with any of the content in this book, you must use the video links included to brush up on these key skills. The videos stated are accessible on www.hegartymaths.com (if you have access to this through your current school) OR www.corbettmaths.com (for everyone).

You are expected to write neatly, show the method clearly.
Diagrams should be drawn using a pencil and ruler.

Content

LAWS OF INDICES

Hegarty Maths No: 102-110

Corbett Maths: 173, 174, 176

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1. (a) Simplify $m^5 \div m^3$

.....

(b) Simplify $5x^4y^3 \times x^2y$

.....

2. Write these numbers in order of size.
Start with the smallest number.

5^{-1} 0.5 -5 5^0

.....

3. Write down the value of $125^{\frac{2}{3}}$

.....

4. (a) Find the value of 5^0

.....

(b) Find the value of $27^{1/3}$

.....

(c) Find the value of 2^{-3}

.....

5. (a) Write down the value of $64^{\frac{1}{2}}$

.....

(b) Find the value of $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

.....

SURDS

Hegarty Maths No: 115-119, 500

Corbett Maths: 305 - 308

---

1. Write $\frac{\sqrt{18}+10}{\sqrt{2}}$ in the form $a+b\sqrt{2}$ where *a* and *b* are integers.

.....

2. (a) Rationalise the denominator of $\frac{1}{\sqrt{3}}$

.....

(b) Expand $(2+\sqrt{3})(1+\sqrt{3})$
Give your answer in the form $a+b\sqrt{3}$ where *a* and *b* are integers.

.....

3. Work out

$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

.....

3. (a) Rationalise the denominator of $\frac{1}{\sqrt{7}}$

.....

(b) (i) Expand and simplify $(\sqrt{3}+\sqrt{15})^2$
Give your answer in the form $a+b\sqrt{3}$ where *a* and *b* are integers.

.....

(ii) All measurements on the triangle are in centimetres.
ABC is a right-angled triangle.
k is a positive integer.

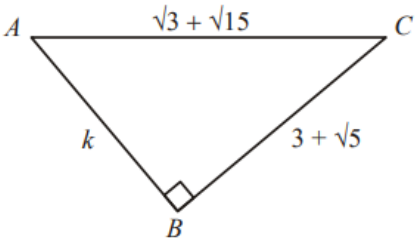


Diagram **NOT** accurately drawn

Find the value of *k*.

k = ...

CHANGING THE SUBJECT FOR A FORMULA

Hegarty Maths No: 280-286

Corbett Maths: 7, 8

---

1. Make u the subject of the formula

$$D = ut + kt^2$$

$u =$ .....

2. Rearrange $(q - c) = d$ to make q the subject.

$q =$ .....

3. Make x the subject of

$$5(x - 3) = (4 - 3x)$$

$x =$ .....

4. Rearrange the formula to make a the subject.

$$P = \frac{n^2 + a}{n + a}$$

$a =$.....

SIMULTANEOUS EQUATIONS (LINEAR AND NON-LINEAR)

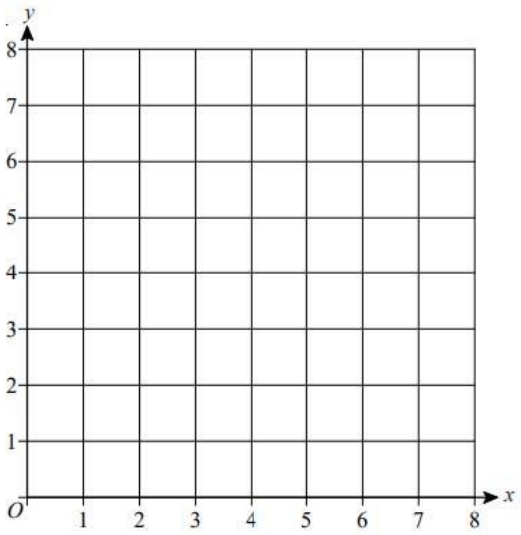
Hegarty Maths No: 192-194, 218-219, 246, 259

Corbett Maths: 295 - 298

---

1. Use the axes provided to solve the simultaneous equations.

$y=2x-1$ and $y=7-x$



2. Solve the simultaneous equations

(a) $3x + 4y = 5$

$2x - 3y = 9$

$x =$ .....

$y =$ .....

(b) $2x + 3y = \frac{2}{3}$

$3x - 4y = 18$

$x =$ .....

$y =$ .....

3. Solve the equations

$$x^2 + y^2 = 36$$
$$x = 2y + 6$$

$x =$ .....

$y =$ .....

4. Solve the equations

$$x + y = 4$$
$$y = x^2 + 3x - 1$$

$x =$ .....

$y =$ .....

EXPANDING AND FACTORISING QUADRATIC EXPRESSIONS/EQUATIONS

Hegarty Maths No: 162-164, 168-169, 223-228, 230-233

Corbett Maths: 118, 119, 119a

---

1. Expand and simplify $(m + 7)(m + 3)$

.....

2. (a) Factorise $6 + 9x$

.....

(b) Factorise $y^2 - 16$

.....

(c) Factorise $2p^2 - p - 10$

.....

3. (a) Factorise $x^2 + 5x + 4$

.....

(b) Expand and simplify $(3x - 1)(2x + 5)$

.....

4. (a) Factorise $x^2 + 7x$

.....

(b) Factorise $y^2 - 10y + 16$

.....

(c) (i) Factorise $2t^2 + 5t + 2$

.....

5. Solve, by factorising, the equation $8x^2 + 2x - 15 = 0$

.....

USING THE QUADRATIC FORMULA

Hegarty Maths No: 241-242

Corbett Maths: 267

---

1. Solve $3x^2 + 7x - 13 = 0$
Give your solutions correct to 2 decimal places.

$x =$ ..... or $x =$ .....

2. Solve the equation

$$2x^2 + 6x - 95 = 0$$

Give your solutions correct to 3 significant figures.

$x =$ ..... or $x =$ .....

3. Solve this quadratic equation. $x^2 = 5x + 8$
Give your answers correct to 3 significant figures.

$x =$ ..... or $x =$ .....

COMPLETING THE SQUARE

Hegarty Maths No: 235-239

Corbett Maths: 267a, 265

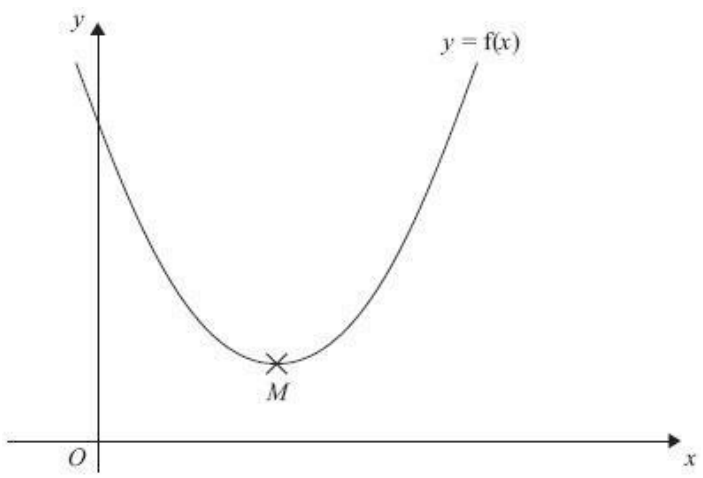
---

1. The expression $x^2 - 8x + 21$ can be written in the form $(x+a)^2 + b$ for all values of x.
(a) Find the value of a and the value of b.

a = ...

b = ...

The equation of a curve is $y = (x)$ where $(x) = x2 - 8x + 21$
The diagram shows part of a sketch of the graph of $y = (x)$.



The minimum point of the curve is M.
(b) Write down the coordinates of M.

.....

2. Sketch the graph of $(x) = x^2 - 5x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

3. (a) Write $2x^2 + 16x + 35$ in the form $a(x+b)^2 + c$ where a,b, and c are integers.

.....

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

.....

INEQUALTIES

Hegarty Maths No: 270-277

Corbett Maths: 178, 179, 182

---

1. Find the range of values of x that is satisfied by the following inequalities below.

You must show your working out.

a) solve $8x - 3 \geq 9$

b) solve $5(3x - 2) \leq 125$

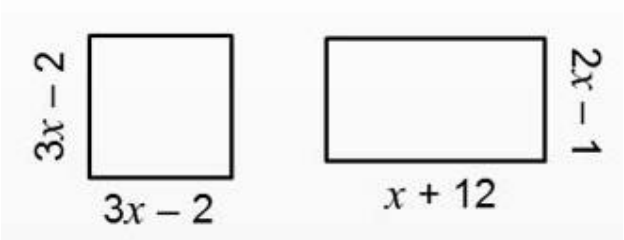
c) solve $\frac{2x - 3}{5} > 9$

d) solve $\frac{3}{8}(5x + 1) \leq 66$

e) solve $-7 \leq 5x + 3 \leq 23$

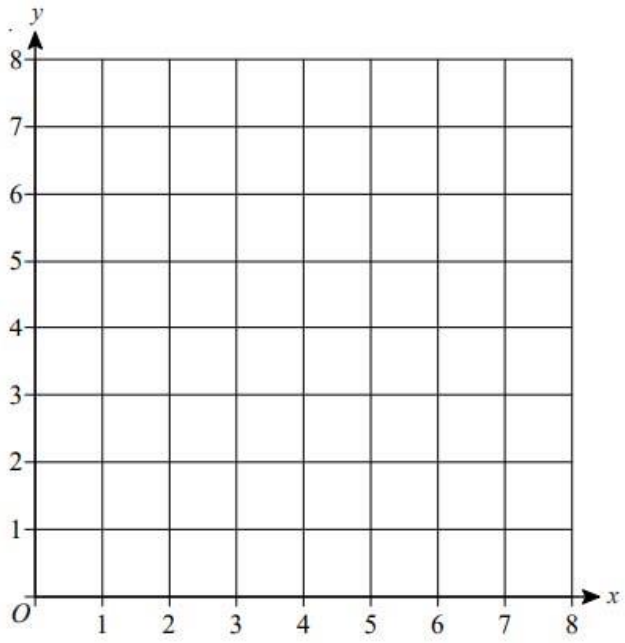
f) solve $1 \leq 6 - 5x \leq 41$

2. For what values of x is the perimeter of the square greater than the perimeter of the rectangle?



3. On the axes provided, draw straight lines, and use shading, to show the region R that satisfies the inequalities detailed.

$$y \geq x+1$$
$$y \geq 5$$
$$x \geq 1$$



4. On the axes provided, draw straight lines, and use shading, to show the region R that satisfies the inequalities detailed.

$$y \geq x$$
$$x+y \leq 7$$
$$x \geq 1$$